

DDPA

A Higher-Order Demand-Driven Program Analysis

Zachary Palmer¹ Scott F. Smith²

Swarthmore College¹

The Johns Hopkins University²

July 20th, 2016

Some Program Analyses

	push forward	demand-driven
first order	abstract interpretation data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA Γ CFA	

Some Program Analyses

	push forward	demand-driven
first order	abstract interpretation data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA Γ CFA	

Some Program Analyses

push forward

demand-driven

first order

abstract interpretation

CFL-reachability

data flow analysis

reverse data flow analysis

higher order

kCFA

CFA2

PDCFA

Γ CFA

Some Program Analyses

	push forward	demand-driven
first order	abstract interpretation data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA Γ CFA	

Some Program Analyses

	push forward	demand-driven
first order	abstract interpretation data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA Γ CFA	DDPA

Some Program Analyses

	push forward	demand-driven
first order	abstract interpretation data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA Γ CFA	DDPA

POLYFLOW_{CFL}

DDPA By Example

```
1 let id x = x;;  
2 let s1 = id 1;;  
3 let s2 = id 2;;
```


DDPA By Example

```
1 let id x = x;;  
2 let s1 = id 1;;  
3 let s2 = id 2;;
```



A-normalize

```
1 id = fun x -> (  
2     ret = x;  
3     );  
4 n1 = 1;  
5 s1 = id n1;  
6 n2 = 2;  
7 s2 = id n2;
```

DDPA By Example

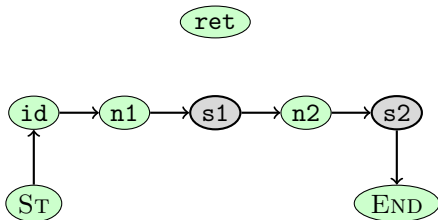
```
1 let id x = x;;  
2 let s1 = id 1;;  
3 let s2 = id 2;;
```



A-normalize

```
1 id = fun x -> (  
2     ret = x;  
3     );  
4 n1 = 1;  
5 s1 = id n1;  
6 n2 = 2;  
7 s2 = id n2;
```

Initial graph before any calls wired:



DDPA By Example

ret



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s1

ret



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s1

Look backward to find function id

ret



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s1

Look backward to find function id

ret



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s1

Look backward to find function id

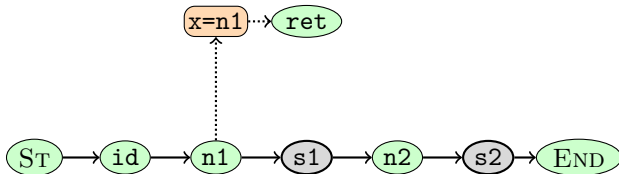


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s1

Bind argument n1 to parameter x

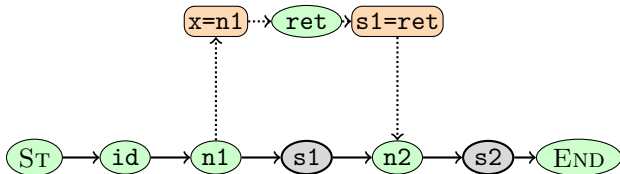


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```


DDPA By Example

Analyze call site s1

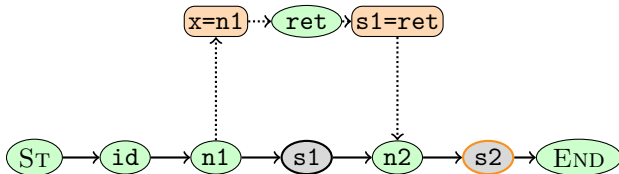
Assign result ret to call site z1



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s2

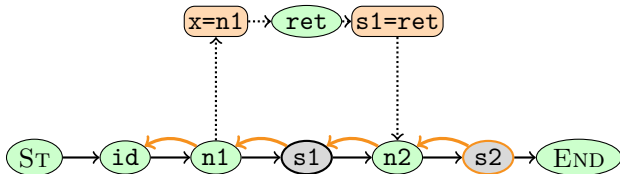


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s2

Look backward to find function id

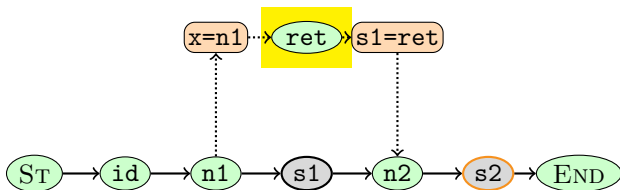


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s2

Look backward to find function id

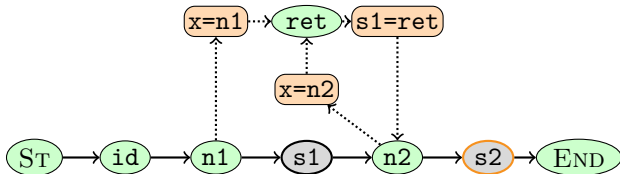


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s2

Bind argument n2 to parameter x

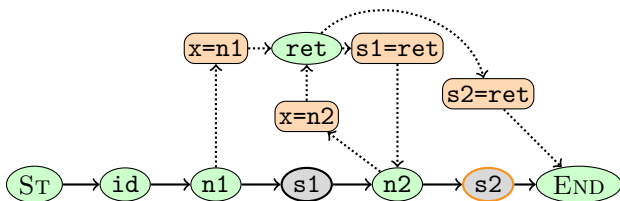


```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

Analyze call site s2

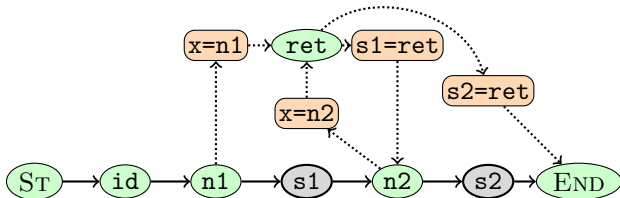
Assign result ret to call site z2



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

DDPA By Example

CFG construction complete



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Observe

- Incrementally built control-flow graph (CFG)

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand
- How could this lookup be accurate? Challenges:

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand
- How could this lookup be accurate? Challenges:
 - Context-sensitivity / polymorphism?
 - Non-local variables?
 - Recursion?
 - Function parameters?
 - State and heap aliases?
 - Path-sensitivity?

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand
- How could this lookup be accurate? Challenges:
 - **Context-sensitivity / polymorphism?**
 - **Non-local variables?**
 - Recursion?
 - Function parameters?
 - State and heap aliases?
 - Path-sensitivity?

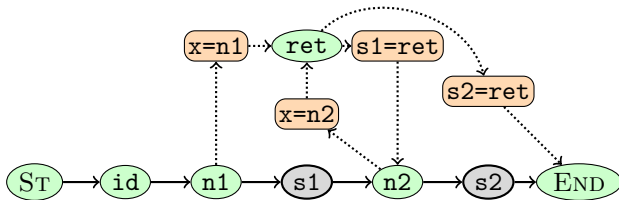
Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand
- How could this lookup be accurate? Challenges:
 - **Context-sensitivity / polymorphism?**
 - **Non-local variables?**
 - Recursion? (**brief mention**)
 - Function parameters?
 - State and heap aliases?
 - Path-sensitivity?

Observe

- Incrementally built control-flow graph (CFG)
- Function bodies were wired to call sites as discovered
- Analysis focused on variable lookup
 - “What values might variable x have at program point p ?”
- Lookup is temporally reversed and on demand
- How could this lookup be accurate? Challenges:
 - **Context-sensitivity / polymorphism?**
 - **Non-local variables?**
 - Recursion? (**brief mention**)
 - Function parameters?
 - State and heap aliases?
 - Path-sensitivity?
 - (Flow-sensitivity comes for free)

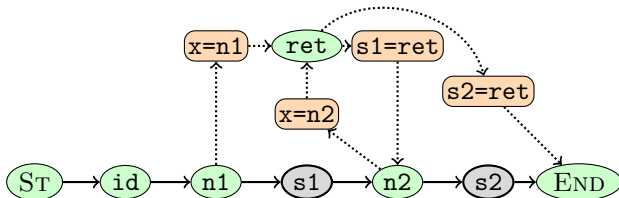
Call Stack Alignment for Context-Sensitivity



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

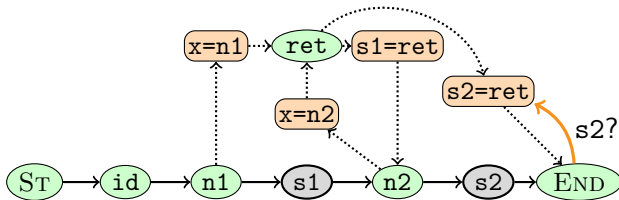
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

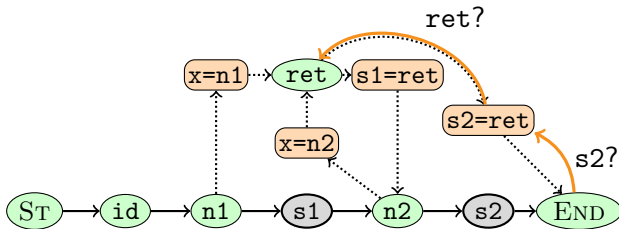
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

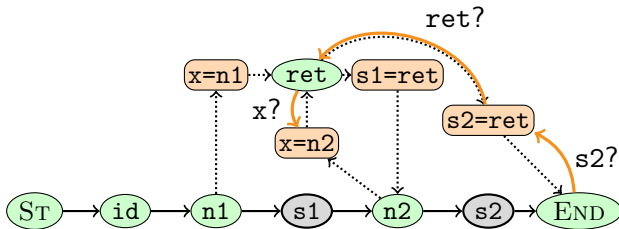
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

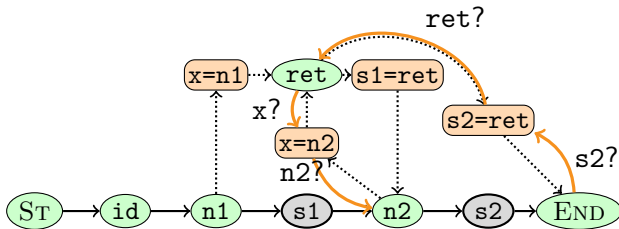
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

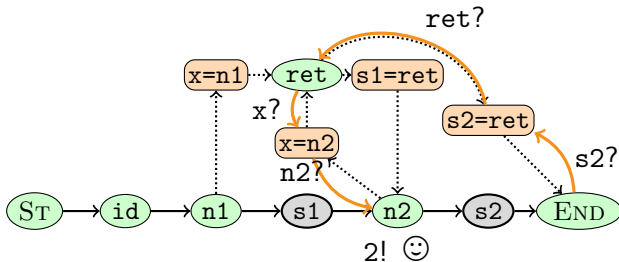
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

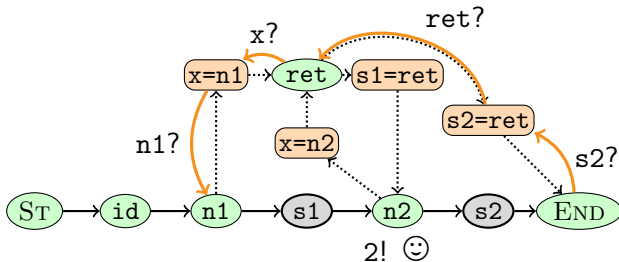
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

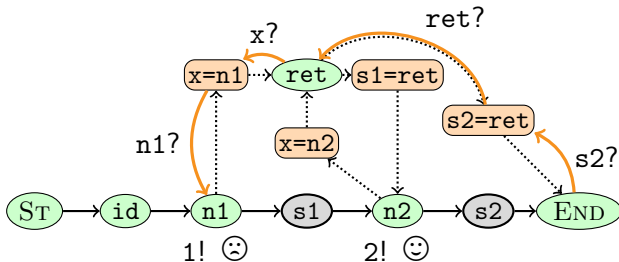
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```


Call Stack Alignment for Context-Sensitivity

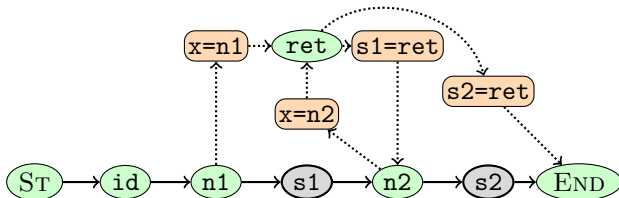
Look up s2 from end of program



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

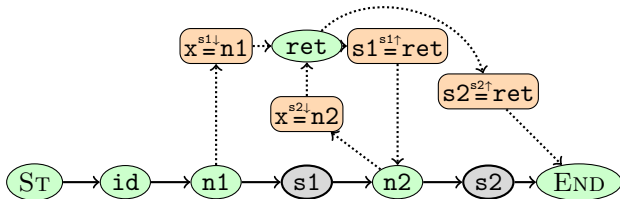
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

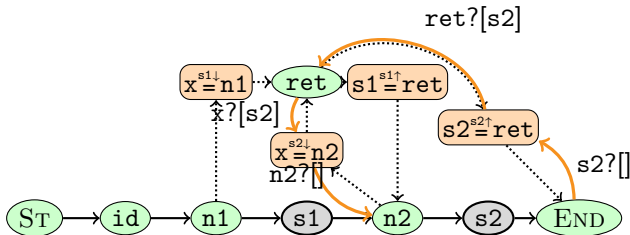
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

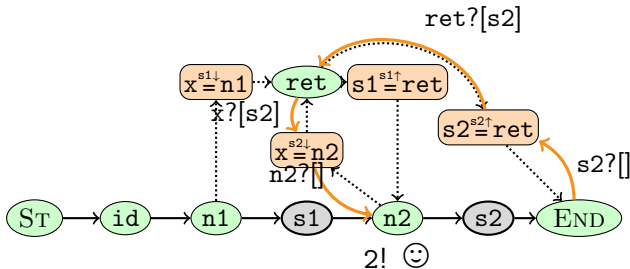
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

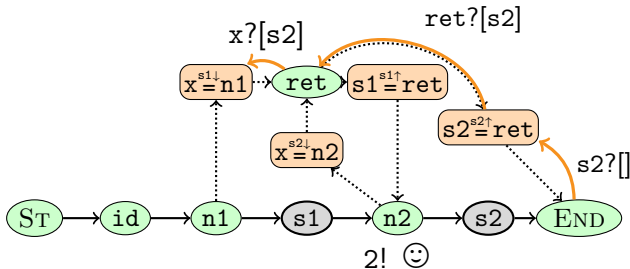
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

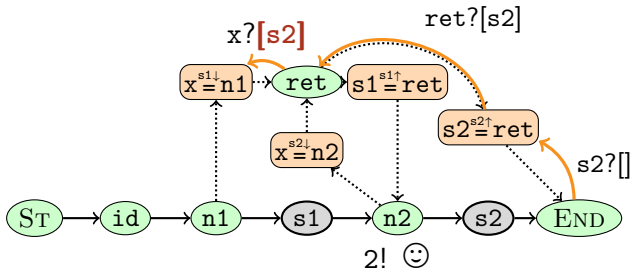
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

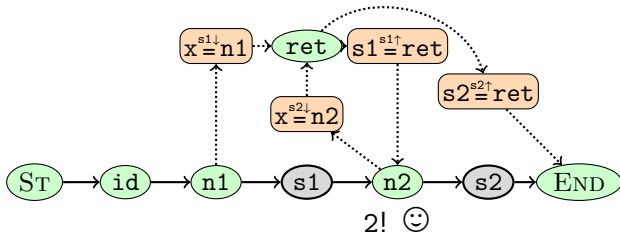
Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```

Call Stack Alignment for Context-Sensitivity

Solution: Maintain call stack during lookup, use to filter



```
1 id = fun x -> ( ret = x; );  
2 n1 = 1;  
3 s1 = id n1;  
4 n2 = 2;  
5 s2 = id n2;
```


Implementing stack alignment

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns
 - Does not align non-locals; uses context copying for that

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns
 - Does not align non-locals; uses context copying for that
- POLYFLOW_{CFL}
 - Uses CFL/PDA for aligning calls and returns

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns
 - Does not align non-locals; uses context copying for that
- POLYFLOW_{CFL}
 - Uses CFL/PDA for aligning calls and returns
 - Not flow-sensitive; lesser precision

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns
 - Does not align non-locals; uses context copying for that
- POLYFLOW_{CFL}
 - Uses CFL/PDA for aligning calls and returns
 - Not flow-sensitive; lesser precision
- DDPA:
 - Also uses PDS: lookup decision \equiv automata reachability

Implementing stack alignment

- CFA2/PDCFA:
 - Push-Down System (PDS) for aligning calls and returns
 - Does not align non-locals; uses context copying for that
- POLYFLOW_{CFL}
 - Uses CFL/PDA for aligning calls and returns
 - Not flow-sensitive; lesser precision
- DDPA:
 - Also uses PDS: lookup decision \equiv automata reachability
 - PDS stack is not call stack
 - We need the PDS stack for something else...

Handling Non-Local Variables

Non-local example: K-combinator

```
1 let k v j = v;;  
2 let f = k 1;;  
3 let g = k 2;;  
4 let s = f 0;;
```


Handling Non-Local Variables

Non-local example: K-combinator

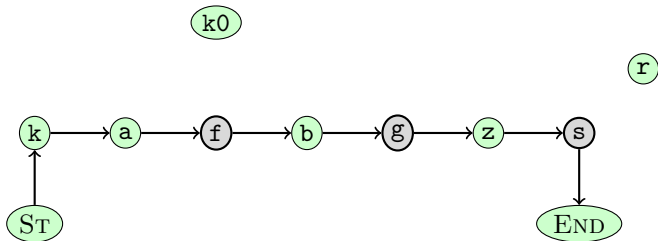
```
1 let k v j = v;;  
2 let f = k 1;;  
3 let g = k 2;;  
4 let s = f 0;;
```



A-normalization

```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup



```
1 k = fun v -> (k0 = fun j -> (r = v;));
```

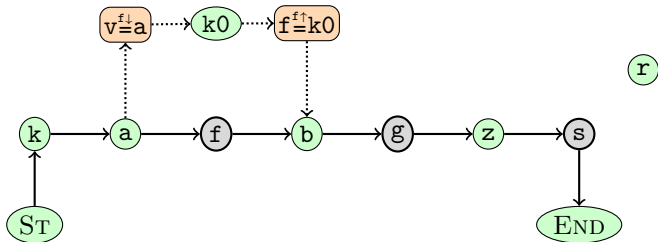
```
2 a = 1; f = k a;
```

```
3 b = 2; g = k b;
```

```
4 z = 0; s = f z;
```

Non-Local Variable Lookup

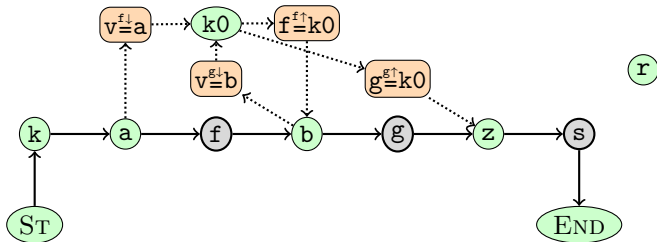
Analyze call site f .



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup

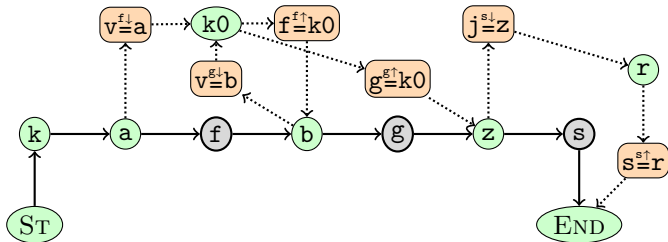
Analyze call site g.



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup

Analyze call site s .



```
1 k = fun v -> (k0 = fun j -> (r = v;));
```

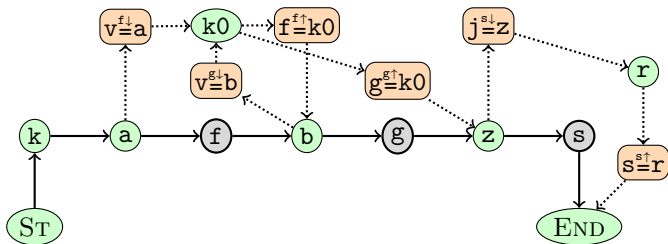
```
2 a = 1; f = k a;
```

```
3 b = 2; g = k b;
```

```
4 z = 0; s = f z;
```

Non-Local Variable Lookup

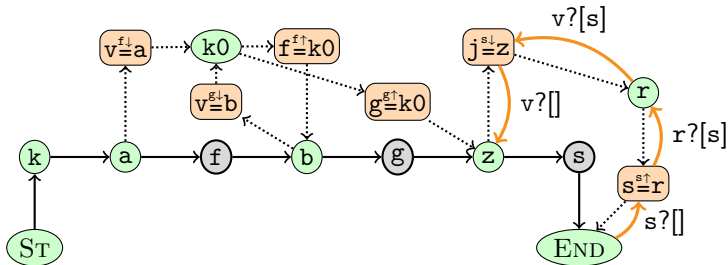
Look up `s` from end of program.



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup

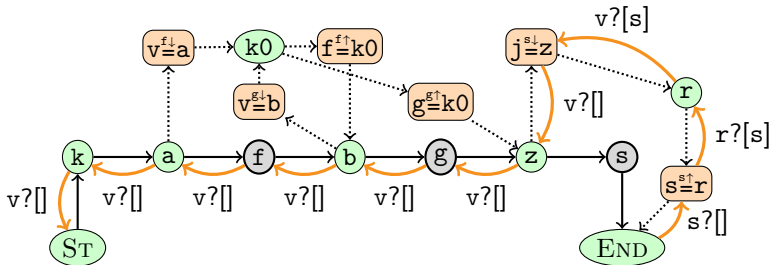
Look up s from end of program.



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup

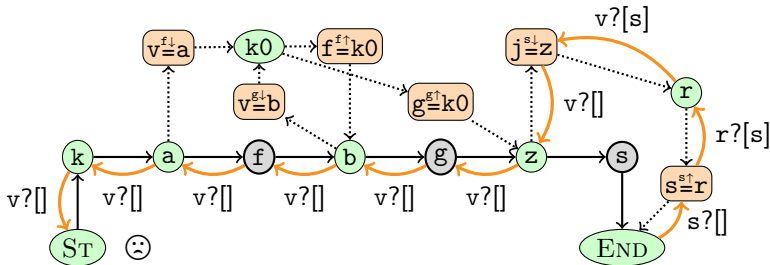
Look up s from end of program.



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

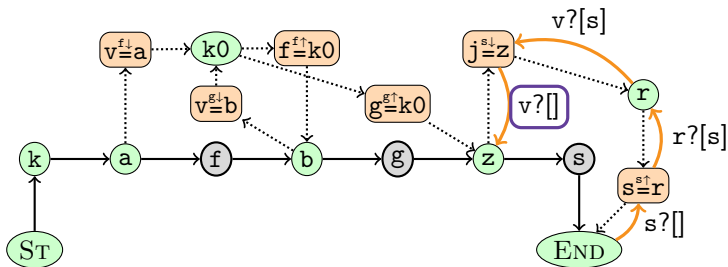

Non-Local Variable Lookup

Look up s from end of program.



```
1 k = fun v -> (k0 = fun j -> (r = v;));  
2 a = 1; f = k a;  
3 b = 2; g = k b;  
4 z = 0; s = f z;
```

Non-Local Variable Lookup

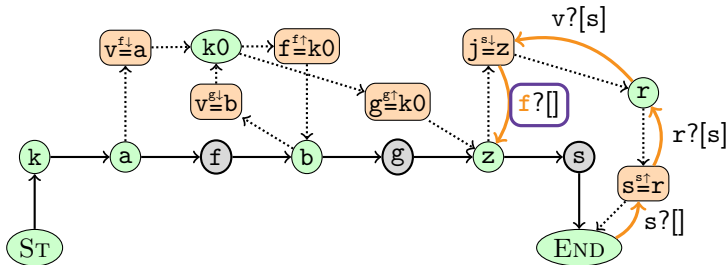


```

1 k = fun v -> (k0 = fun j -> (r = v;));
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Non-Local Variable Lookup

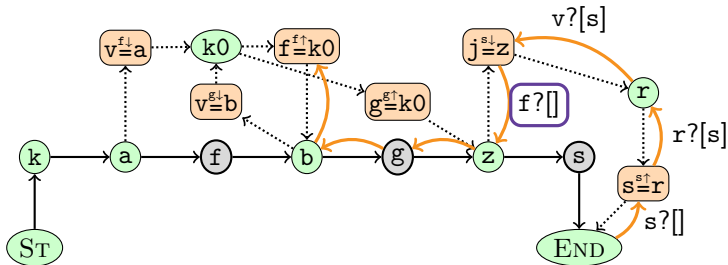


```

1 k = fun v -> (k0 = fun j -> (r = v;)););
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Non-Local Variable Lookup

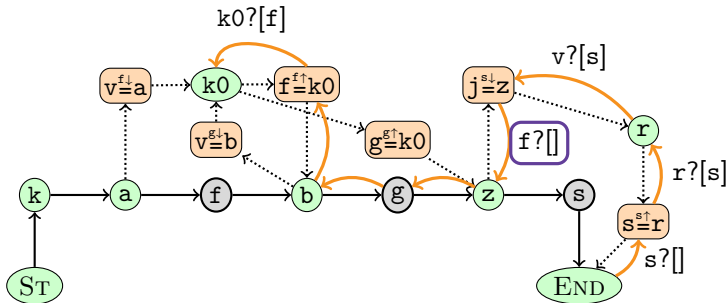


```

1 k = fun v -> (k0 = fun j -> (r = v;)););
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Non-Local Variable Lookup

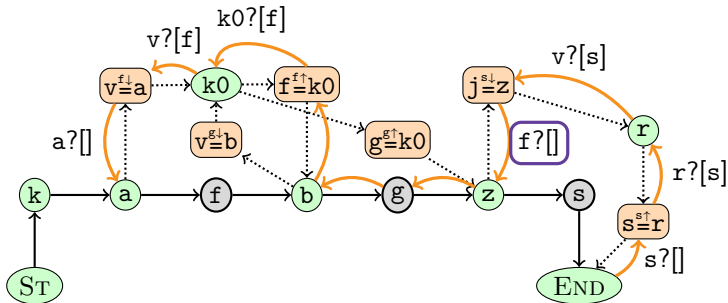


```

1 k = fun v -> (k0 = fun j -> (r = v;));
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Non-Local Variable Lookup

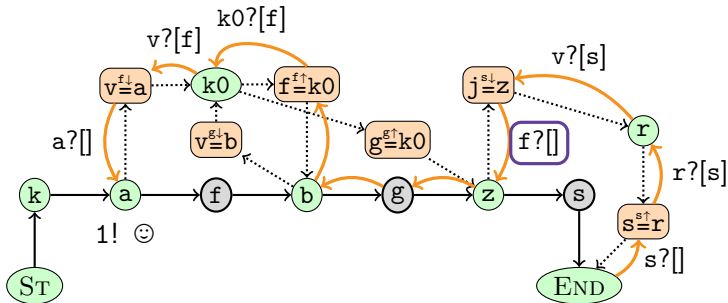


```

1 k = fun v -> (k0 = fun j -> (r = v;)););
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Non-Local Variable Lookup



```

1 k = fun v -> (k0 = fun j -> (r = v;)););
2 a = 1; f = k a;
3 b = 2; g = k b;
4 z = 0; s = f z;

```

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable
- General case: continuation stack needed for non-local lookups

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable
- General case: continuation stack needed for non-local lookups
- Can't have a 2-stack PDS!

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable
- General case: continuation stack needed for non-local lookups
- Can't have a 2-stack PDS!
 - Solution: finitize call stack in PDS nodes; keep full lookup stack.

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable
- General case: continuation stack needed for non-local lookups
- Can't have a 2-stack PDS!
 - Solution: finitize call stack in PDS nodes; keep full lookup stack.
 - k DDPA: maximum call stack depth k

Handling Non-Local Variables

- Search for defining closure; then, resume looking for variable
- General case: continuation stack needed for non-local lookups
- Can't have a 2-stack PDS!
 - Solution: finitize call stack in PDS nodes; keep full lookup stack.
 - k DDPA: maximum call stack depth k
 - Lookup still translates to PDS reachability decision problem

Properties

- Recursion: handled by PDS lookup; cycles are fine in a PDS

Properties

- Recursion: handled by PDS lookup; cycles are fine in a PDS
- Theorem: k DDPA (for fixed k) has polynomial time bound

Properties

- Recursion: handled by PDS lookup; cycles are fine in a PDS
- Theorem: k DDPA (for fixed k) has polynomial time bound
- Lemma: both CFG and PDS are monotonically increasing over analysis

Properties

- Recursion: handled by PDS lookup; cycles are fine in a PDS
- Theorem: k DDPA (for fixed k) has polynomial time bound
- Lemma: both CFG and PDS are monotonically increasing over analysis
 - Allows for analysis to be purely additive – efficient sharing

Properties

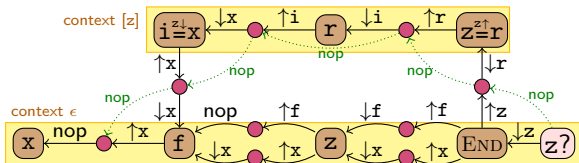
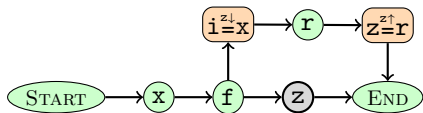
- Recursion: handled by PDS lookup; cycles are fine in a PDS
- Theorem: k DDPA (for fixed k) has polynomial time bound
- Lemma: both CFG and PDS are monotonically increasing over analysis
 - Allows for analysis to be purely additive – efficient sharing
 - Observe we have reduced program analysis to incremental (PDS) model checking - fast!

Source / CFG / PDS - the whole analysis

```

1 x = {};
2 f = fun i -> ( r = i );
3 z = f x;

```



- Build both CFG and PDS incrementally; above is final result

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics
- DDPA is demand-driven and model-checking: potentially more efficient than forward analyses

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics
- DDPA is demand-driven and model-checking: potentially more efficient than forward analyses
- k in k DDPA not directly comparable to k CFA, etc.

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics
- DDPA is demand-driven and model-checking: potentially more efficient than forward analyses
- k in k DDPA not directly comparable to k CFA, etc.
 - k of k DDPA also needs to be bigger for handling non-locals

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics
- DDPA is demand-driven and model-checking: potentially more efficient than forward analyses
- k in k DDPA not directly comparable to k CFA, etc.
 - k of k DDPA also needs to be bigger for handling non-locals
 - Which makes sense: for fixed k , k CFA is EXPTIME but k DDPA is polynomial

Forward and reverse analyses compared

- Forward analyses are more natural to derive from operational semantics
- DDPA is demand-driven and model-checking: potentially more efficient than forward analyses
- k in k DDPA not directly comparable to k CFA, etc.
 - k of k DDPA also needs to be bigger for handling non-locals
 - Which makes sense: for fixed k , k CFA is EXPTIME but k DDPA is polynomial
- Practically speaking, expressiveness appears similar

Implementation

- Paper artifact: inefficient proof-of-concept

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features
 - Records

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features
 - Records
 - Path-sensitivity – paths validated by PDS

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features
 - Records
 - Path-sensitivity – paths validated by PDS
 - Heap-sensitive state including may/must alias information

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features
 - Records
 - Path-sensitivity – paths validated by PDS
 - Heap-sensitive state including may/must alias information
- Lazily constructs PDS according to regular definition

Implementation

- Paper artifact: inefficient proof-of-concept
- Now: efficient implementation with additional features
 - Records
 - Path-sensitivity – paths validated by PDS
 - Heap-sensitive state including may/must alias information
- Lazily constructs PDS according to regular definition
- Looks to be reasonably efficient

Future Work

- Variable alignment for precision

Future Work

- Variable alignment for precision
- Better call stack model for performance

Future Work

- Variable alignment for precision
- Better call stack model for performance
- Application to existing languages

Conclusions

- DDPA: first flow-sensitive, demand-driven, higher-order program analysis
- Program analysis based on incremental model checking
 - Promising for efficiency
- Appears comparable in expressiveness with state-of-the-art forward analyses
- Code: <https://github.com/JHU-PL-Lab/odefa>