# Control-Based Program Analysis 

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November 23rd, 2015

## Outline

- CBA Overview
- CBA by Example
- Properties of CBA
- Comparison to Related Analyses
- Implementation
- Conclusions


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- All values looked up relative to point in CFG
- Relative lookup yields flow-sensitive analysis
- CFG is the only data structure
- No abstract environment or store
- So, variable lookup only needs CFG


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## A Very Simple Example

$$
\begin{aligned}
& 1 \text { let id } x=x ; ; \\
& 2 \text { let } s 1=\text { id } 1 ; \\
& 3 \text { let } s 2=\text { id } 2 ;
\end{aligned}
$$

## A Very Simple Example

```
1 let id x = x;;
2 let s1 = id 1;;
3 let s2 = id 2;;
    | A-normalization
1 id = fun x -> (
2 ret = x;
3 );
4 n1 = 1;
5 s1 = id n1;
6 n2 = 2;
7 s2 = id n2;
```


## A Very Simple Example



## A Very Simple Example

## Graph closure

```
ret
```



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## A Very Simple Example

## Graph closure for call site s1

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1 id \(=\) fun \(x \rightarrow\) (ret \(=x ;)\);
\(2 \mathrm{n} 1=1\);
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Look backward to find function id


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## ret



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## Graph closure for call site s1

Bind argument n 1 to parameter x


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## Graph closure for call site s1

Assign result ret to call site z1


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## Graph closure for call site s2



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Bind argument n 2 to parameter x


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## Graph closure for call site s2

Assign result ret to call site z2


## A Very Simple Example

Closure complete!


## Lookup: Related Work

- Lookup is temporally reversed and on demand


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- Non-local variables


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## Goal: polymorphism



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Look up s2 from end of program


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Annotate wiring nodes with call sites


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## Call Stack Alignment

## We need to match calls and returns.

Here, 1 is eliminated


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- CFL-reachability analyses: calls and returns modeled as CFL
- CFA2 [VS-ESOP10] and PDCFA [MSV-PLDI10]: align calls and returns via PDA
- PDA is precisely an abstract interpreter


# Handling Non-Local Variables 

Non-local example: K-combinator

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1 let k v j = v;;
2 let f = k 1;;
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4 let s = f 0;;
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& 3 \text { let } \mathrm{g}=\mathrm{k} 2 \text {; ; } \\
& 4 \text { let } \mathrm{s}=\mathrm{f} 0 ; \text {; } \\
& \Downarrow \text { A-normalization } \\
& 1 \mathrm{k}=\text { fun } \mathrm{v} \rightarrow \text { ( } \mathrm{k} 0=\text { fun } \mathrm{j} \rightarrow \text { ( } \mathrm{r}=\mathrm{v} ; \text { ) ; ) ; } \\
& \text { 2 } \mathrm{a}=1 ; \quad \mathrm{f}=\mathrm{k} \mathrm{a} \text {; } \\
& 3 \mathrm{~b}=2 ; \mathrm{g}=\mathrm{k} \mathrm{~b} \text {; } \\
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\end{aligned}
$$

## Handling Non-Local Variables

## Perform closure


$1 \mathrm{k}=$ fun $\mathrm{v} \rightarrow$ ( $k 0=$ fun $j \rightarrow(\mathrm{r}=\mathrm{v} ;)$; ) ;
$2 \mathrm{a}=1 ; \quad \mathrm{f}=\mathrm{k} \mathrm{a}$;
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## Handling Non-Local Variables

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...for call site f.


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Look up s from end of program.

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- $k$ CBA: maximum call stack depth $k$


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- Therefore, graph size $g$ is $O\left(n^{2}\right)$


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- Delightful mathematical property; huge win for optimization!


## Outline

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- CBA by Example
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- Comparison to Related Analyses
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- $(k+c)$ CBA strictly more expressive than $k$ CFA


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- Polyvariance
- PDCFA: classic CFA-like graph copying
- CBA: via call stack alignment and non-local lookup


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- Path-sensitivity model: possible theorem-proving applications


## Questions?

- Code: https://github.com/JHU-PL-Lab/odefa-proof-of-concept
- Paper: http://pl.cs.jhu.edu/projects/big-bang/papers/ control-based-program-analysis.pdf


## Example of $k$ CBA Imprecision

- Consider code:

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\begin{aligned}
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& 2 \text { let } \mathrm{x} y=\mathrm{y} y ; \\
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- 2CBA: $\mathrm{a} \subseteq\{1\}$
- Alternative CBA call stack finitizations exist (e.g. regex)


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- Such as used in pushdown-assisted CFA

