Control-Based Program Analysis

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Outline

- CBA Overview
- CBA by Example
- Properties of CBA
- Comparison to Related Analyses
- Implementation
- Conclusions

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 - So, variable lookup only needs CFG

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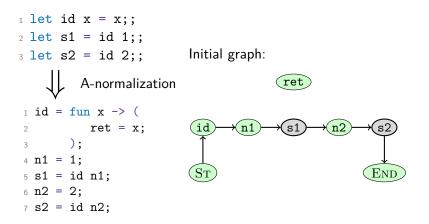
A Very Simple Example

1 let id x = x;; 2 let s1 = id 1;; 3 let s2 = id 2;;

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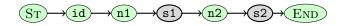
```
1 let id x = x;;
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       A-normalization
_1 id = fun x -> (
2 ret = x;
3 );
4 n1 = 1;
5 s1 = id n1;
6 n2 = 2;
7 \text{ s2} = \text{id n2};
```

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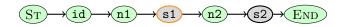
A Very Simple Example Graph closure





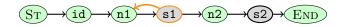
A Very Simple Example Graph closure for call site s1





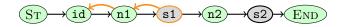
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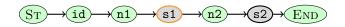
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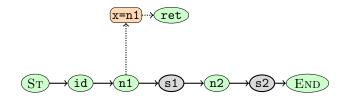


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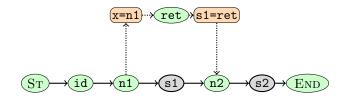




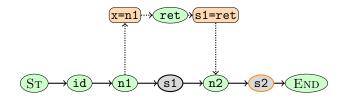
A Very Simple Example Graph closure for call site s1 Bind argument n1 to parameter x



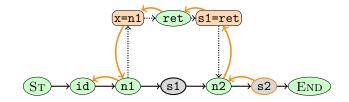
A Very Simple Example Graph closure for call site s1 Assign result ret to call site z1



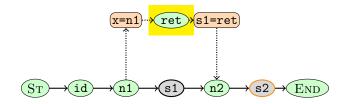
A Very Simple Example Graph closure for call site s2



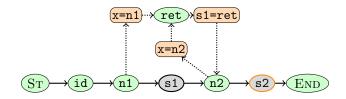
A Very Simple Example Graph closure for call site s2 Look backward to find function id



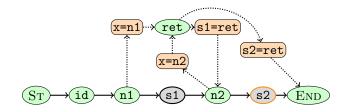
A Very Simple Example Graph closure for call site s2 Look backward to find function id



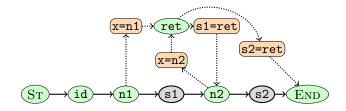
A Very Simple Example Graph closure for call site s2 Bind argument n2 to parameter x



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A Very Simple Example Closure complete!



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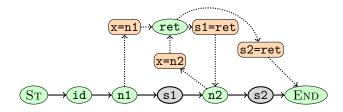
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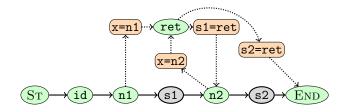
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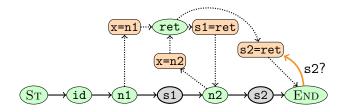
Lookup: Related Work

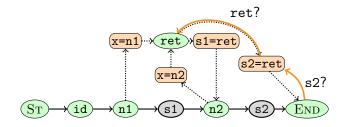
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 - Polyvariance
 - Non-local variables

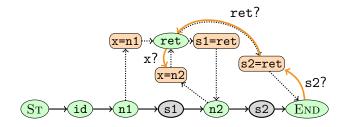
Call Stack Alignment Goal: polymorphism

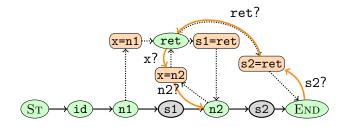


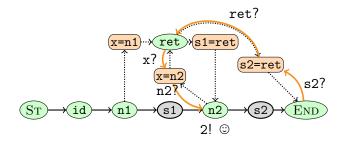


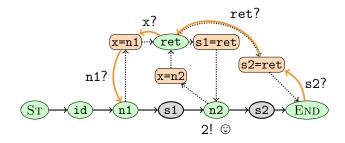


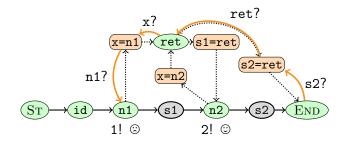




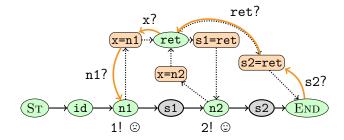




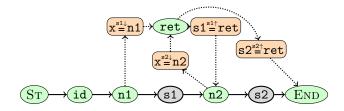


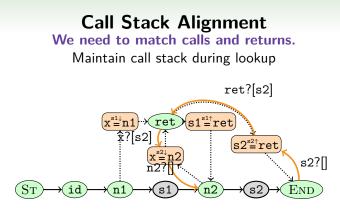


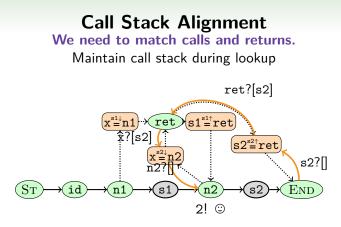
Call Stack Alignment We need to match calls and returns.

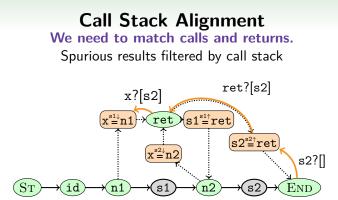


Call Stack Alignment We need to match calls and returns. Annotate wiring nodes with call sites



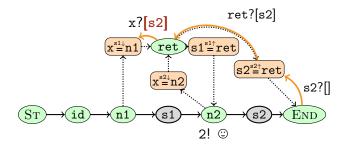






2! 🙂

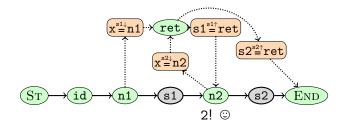




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We need to match calls and returns.

Here, 1 is eliminated



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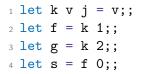
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- CFA2 [VS-ESOP10] and PDCFA [MSV-PLDI10]: align calls and returns via PDA
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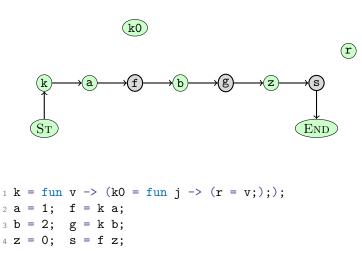
Handling Non-Local Variables

Non-local example: K-combinator



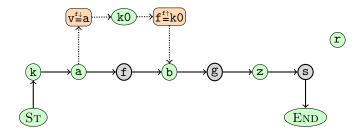
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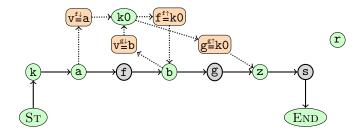


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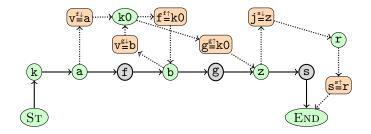
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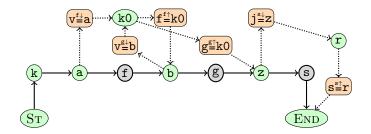


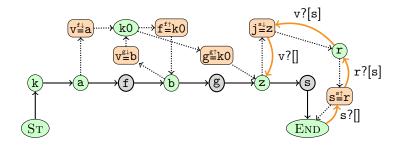
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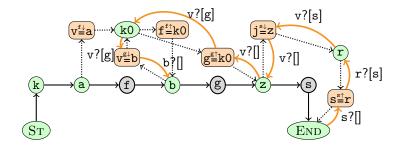


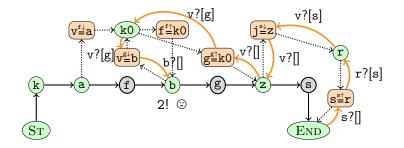
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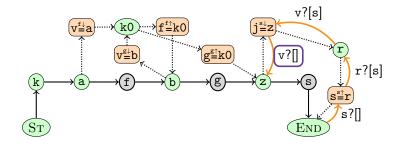


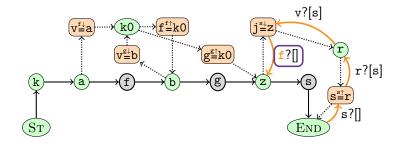


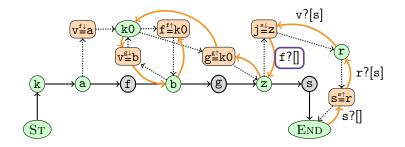




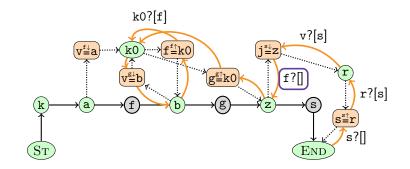




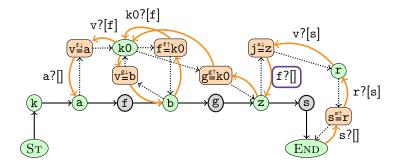




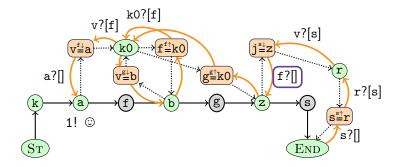
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 - kCBA: maximum call stack depth k

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 - Delightful mathematical property; huge win for optimization!

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 - ${\scriptstyle \bullet}\,$ Suppose program with max lexical nesting depth c
 - (k + c)CBA strictly more expressive than kCFA

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Polyvariance

- PDCFA: classic CFA-like graph copying
- CBA: via call stack alignment and non-local lookup

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 - Uses monotonicity lemma: same lazy PDA for all lookups

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 - Path-sensitivity model: possible theorem-proving applications

Questions?

- Code: https://github.com/JHU-PL-Lab/odefa-proof-of-concept
- Paper: http://pl.cs.jhu.edu/projects/big-bang/papers/ control-based-program-analysis.pdf

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 - 2CBA: $a \subseteq \{1\}$
 - Alternative CBA call stack finitizations exist (e.g. regex)

- Consider code:
- 1 let f x = x;;
- 2 let g y = f y;;
- 3 let a = g 1;;
- 4 let b = g 2;;
 - 1CBA: $\mathtt{a} \subseteq \{\mathtt{1},\mathtt{2}\}$
 - From within f, we can't remember where g was called
 - 1CFA: same problem
 - 2CBA: $a \subseteq \{1\}$
 - Alternative CBA call stack finitizations exist (e.g. regex)
 - Such as used in pushdown-assisted CFA