

Type Systems

- related to type checking
- ascribes to each part of a program a property called a "type"
- types include descriptions of data representations (type is a set of values)*

- a set is an unordered collection of unique values

$$\begin{array}{ll} \{1, 2\} & \{3, 4, 5\} \\ \overline{\{3\}} & \{6, 11\} \end{array}$$

\downarrow

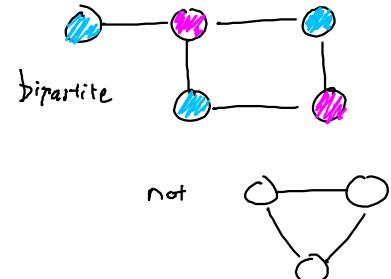
$$\begin{array}{l} \{3, 3\} = \{3\} \\ \{3, 4\} = \{4, 3\} \end{array}$$

→ $\text{bool} = \{\text{true}, \text{false}\}$

$$\text{int} = \{0, 1, -1, 2, -2, \dots, 2^{62}-1, -2^{62}\}$$

bipartite tree

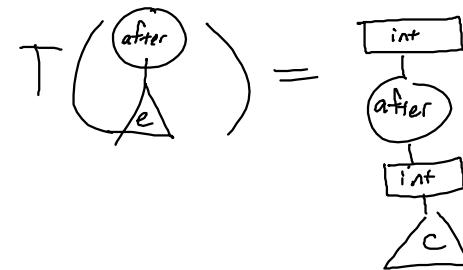
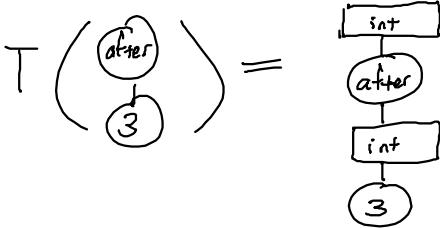
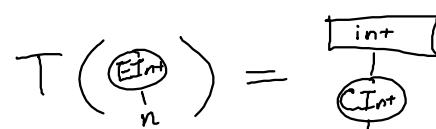
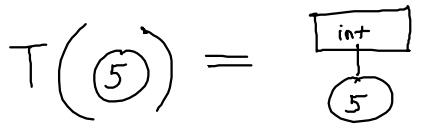
```
type cexpr =
| C Int of int
| C After of checked
...
| C Checked of cexpr * typ
```



and checked =

| Checked of cexpr * typ

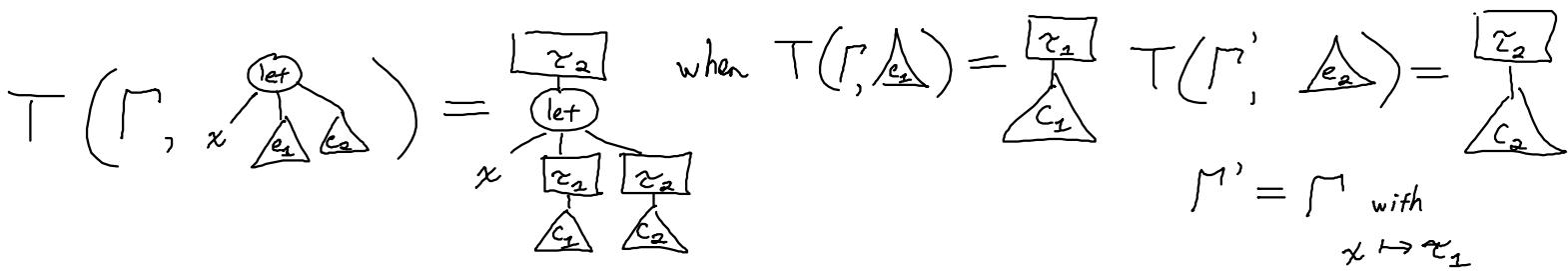
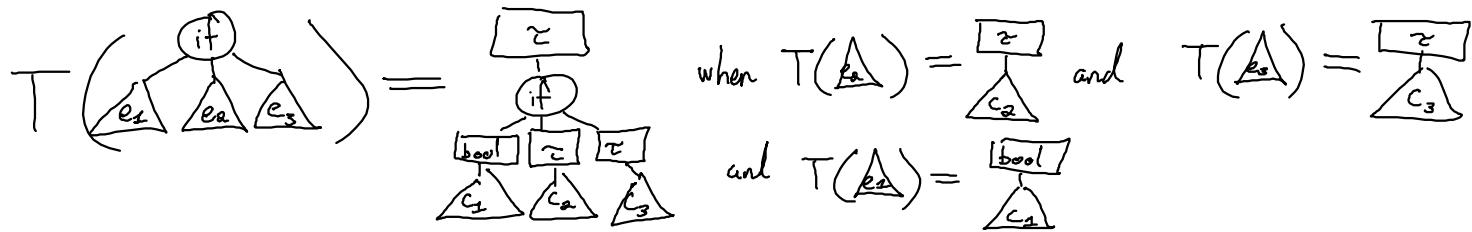
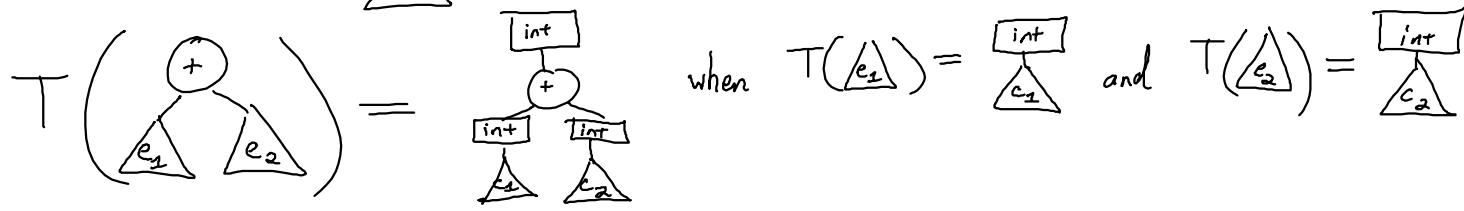
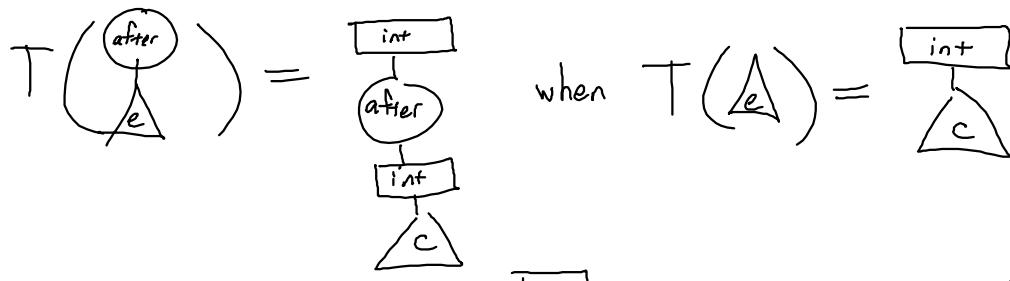
$T: \text{expr} \rightarrow \text{checked}$



a checked ast will, when evaluated, produce a value in the set that is the indicated type (if it evaluates)

when $T(A) =$

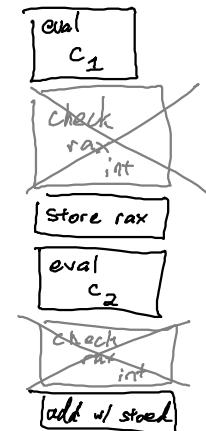
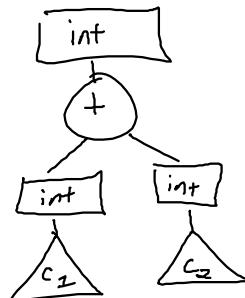
A type tree where the root is an 'int' box, which points to a 'c' triangle.



$$T(\Gamma, x) = \begin{array}{c} \gamma \\ | \\ x \end{array}$$

when $\Gamma(x) = \gamma$

Why?



1. Help programmer understand code & find mistakes
2. Help software (like compiler) understand code (optimization)