

$$1 \frac{\quad}{\Delta} \quad 2 \frac{\Delta s}{\Delta s} \quad 3 \frac{\Delta}{\square}$$

$$S \Rightarrow S$$

$$e \Rightarrow v$$

$$\forall e. \exists v. e \Rightarrow v$$

$$1 \frac{\quad}{\Delta} \\ 2 \frac{\Delta}{\Delta} \\ 3 \frac{\Delta}{\square}$$

Fb

interpreter : source  $\xrightarrow{\text{behavior}}$  value  
 compiler : source  $\rightarrow$  computer-readable

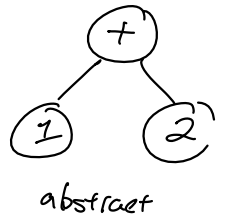
$e ::= v \mid e \text{ Or } e \mid \dots \mid \text{Not } e \mid e + e \mid e - e \mid e \text{ e} \mid \text{If } e \text{ Then } e \text{ Else } e$

$v ::= \text{True} \mid \text{False} \mid 0 \mid 1 \mid -1 \mid 2 \mid -2 \mid \dots \mid \text{Function } x \rightarrow e$

$x ::= (\text{variables})$

Define:  $\text{eval}^{(gst)}(\llbracket e \rrbracket) = \llbracket v \rrbracket$   
 iff  
 $e \Rightarrow v$

$1 + 2$   
 concrete



# Fb Operational Semantics $e \Rightarrow v$

~~2 "apple"~~

$$\overline{v \Rightarrow v}$$

$$\checkmark 4 \Rightarrow 4$$

$$\checkmark \text{True And False} \Rightarrow \text{False}$$

$$\checkmark \text{True And (False Or False)} \Rightarrow \text{False}$$

And (False right)  $\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow \text{False} \quad v_1 \in \{\text{True}, \text{False}\}}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$

And (generic)  $\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v \text{ is "logical and" of } v_1 \text{ and } v_2}{e_1 \text{ And } e_2 \Rightarrow v}$   $\checkmark (1 + \text{False}) \text{ And False} \Rightarrow \text{False}$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1, v_2 \in \{\text{True}, \text{False}\} \quad \text{False} \in \{v_1, v_2\}}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

$$e \not\Rightarrow v$$

for  $e$ ,  $\nexists v$  st.  $e \Rightarrow v$   
"stuck"

converge if  $e \Rightarrow v$   
diverge if  $e \not\Rightarrow v$

Plus (generic)  $\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1, v_2 \in \mathbb{Z} \quad v \text{ is sum of } v_1 \text{ and } v_2}{e_1 + e_2 \Rightarrow v}$

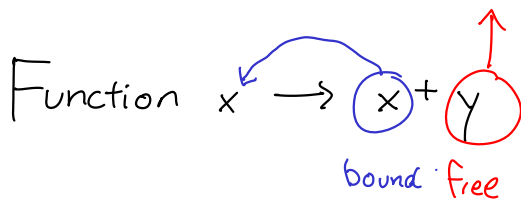
Function  $x \rightarrow e$

let  $f \ x = x$

let  $f = \text{fun } x \rightarrow x$

$$(\text{Function } x \rightarrow x+1) \ 4 \rightarrow 4+1 \Rightarrow 5$$

Variables can be bound or free

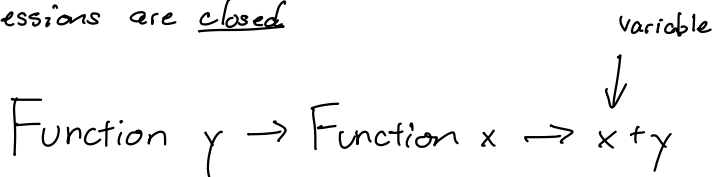


$$e \Rightarrow v$$

is defined only for closed  $e$

An expression w/ free variables is open.

Other expressions are closed.



$$e + e \leftarrow \text{metavariable}$$

$$\text{Function } x \rightarrow e \leftarrow \text{metavariable}$$

$$x \not\Rightarrow$$

$$\left( \text{Function } x \rightarrow \underbrace{x+1}_{4/x} \right) 4$$

$$e[v/x]$$

$$\left( \text{Function } x \rightarrow x+1 \right) [4/x]$$

$$\left( \text{Function } a \rightarrow \underbrace{\text{Function } b \rightarrow a+b}_{[5/a]} \right) 5$$

$$\text{Function } b \rightarrow 5+b$$

$$\left( \text{Function } a \rightarrow \text{Function } a \rightarrow \underbrace{a+a}_{\text{blue circles}} \right) 5$$

$$\text{Function } a \rightarrow a+a$$

$$n[v/x] = n$$

$$x[v/x] \doteq v$$

$$\text{True}[v/x] = \text{True}$$

$$(e_1 + e_2)[v/x] =$$

$$e_1[v/x] + e_2[v/x]$$



$$\left( \text{Function } x' \rightarrow e \right) [v/x] =$$

$$\text{Function } x' \rightarrow e[v/x]$$

$$x \neq x'$$

$$\left( \text{Function } x \rightarrow e \right) [v/x] =$$

$$\left( \text{Function } x \rightarrow e \right)$$

