FbS — Fb with state

$S ::= \{c \mapsto v, \ldots\}$

$c ::=$ (countably infinite set)

$\langle S, e \rangle \Longrightarrow \langle S, v \rangle$

$$\frac{\langle S_0, e_1 \rangle \Longrightarrow \langle S_1, v_1 \rangle \qquad \langle S_1, e_2 \rangle \Longrightarrow \langle S_2, v_2 \rangle}{\langle S_0, e_1 \text{ And } e_2 \rangle \Longrightarrow \langle S_2, \ldots \rangle}$$

$\forall v': (c \mapsto v') \notin S_1$

$$\frac{\langle S_0, e \rangle \Longrightarrow \langle S_1, v \rangle \qquad c \notin S_1 \qquad S_2 = S_1 \cup \{c \mapsto v\}}{\langle S_0, \text{Ref } e \rangle \Longrightarrow \langle S_2, c \rangle}$$

$3 \notin \{4, 5, 6\}$

$3 \notin \{(4,5), (6,7)\}$

$3 \notin \{(3,3)\}$

$\{\#1 \mapsto 4,$
$\#2 \mapsto \text{False},$
$\#3 \mapsto \text{Fun } a \to a\}$

```
let result1 = f 4 in
let result2 = f 4 in
...


let (result1, next_fresh_var_2) = f (next_fresh_var_1) 4 in
let (result2, next_fresh_var_3) = f (next_fresh_var_2) 4 in
...
```

What to do induction on?

- expression?
- proof?

given

produce

TFb Soundness: If $\Gamma \vdash e : \tau$ and $e \Rightarrow v$ then $\Gamma \vdash v : \tau$.

because $e \Rightarrow v$,
e might not get smaller,
so induction on e unlikely to work

given

produce

BOOL normalization: $\forall e.\ \exists v.\ e \Rightarrow v.$

must induct e b/c it's all we have

Good default: on e.

Operational equivalence: $\cong$

$$4+1 \cong 3+2$$

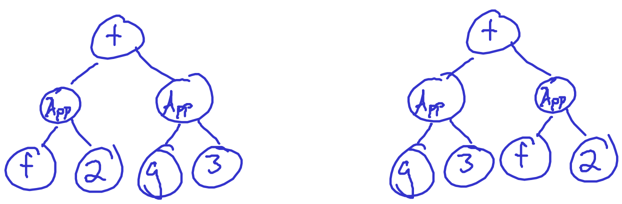
$$4+1 \Downarrow 5 \qquad 3+2 \Downarrow 5$$

Context C is an expression containing exactly one hole $\bullet$

$$e_1 \cong e_2 \text{ iff } \forall C.\ C[e_1] \Rightarrow v_1 \text{ iff } C[e_2] \Rightarrow v_2$$

Fb

$$f\ 2 + g\ 3 \cong g\ 3 + f\ 2$$




```
Let b = Ref False In
Let f = Function a ->
    If a = 2 Then
        b := True
```

FbS

$$f\ 2 + g\ 3 \not\cong g\ 3 + f\ 2$$

C

```
Let c = Ref 0 In
Let f = Function a -> !c In
Let g = Function a -> c := 5 In
Let n = • In
If n = 5 Then 4 4 Else 5
```

FbX

$$f(2) + g(3) \not\cong g(3) + f(2)$$

```
Try
  Let f = Fun a -> Raise (#A 0) In
  Let g = Fun a -> Raise (#B 0) In
  •
With
  #A n -> 4 4
```

$$\frac{e_1 \Rightarrow v_1 \qquad e_2 \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Longrightarrow v_2}$$

Let $x=3$ In 4

$$\frac{3 \Rightarrow 3 \qquad x+4 \Rightarrow}{\text{Let } x=3 \text{ In } x+4}$$