Review: proof of BOOL normalization

$$\underline{\forall e}\ \exists v.\ e \Rightarrow v$$

Without loss of generality (wlog)

Strong Induction on height of expression.

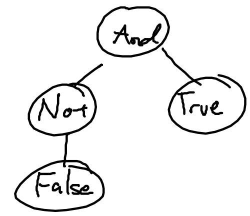
Case analysis on expression form.

Natural induction.

$P(0)$. $P(k)$ implies $P(k+1)$.

Natural strong induction

$P(0)$. $P(0) \land P(1) \land \ldots \land P(k)$ implies $P(k+1)$.



BOOL is deterministic

$\forall e, v_1, v_2 .\ e \Rightarrow v_1$ and $e \Rightarrow v_2$ implies $v_1 = v_2$.

By induction on height of e.

Base case: height e is 0. So e is either True or False. So the proof of $e \Rightarrow v_1$ uses the Value rule. Thus, $e = v_1$. Likewise $e = v_2$. By transitivity, $v_1 = v_2$. ✓

Inductive step: height e is k+1. So e is of form Not e' or e_1 And e_2 or e_1 Or e_2. By case analysis on form of expression:

- If e = Not e' for some e' then e' has height k. We have $e \Rightarrow v_1$. This proof must use the Not rule. So for some v_1', $e' \Rightarrow v_1'$. Also, v_1' is the log. neg. of v_1. We also have $e \Rightarrow v_2$. So for some v_2', $e' \Rightarrow v_2'$. Also v_2' is the log neg of v_2. By ind hyp and because $e' \Rightarrow v_1'$ and $e' \Rightarrow v_2'$ then $v_1' = v_2'$. By logical negation, $v_1 = v_2$. ✓
- If $e = e_1$ And e_2 for some e_1 and e_2, then e_1 and e_2 have heights $\leq k$. We have $e \Rightarrow v_1$. This proof must use the And rule. So $e_1 \Rightarrow v_1'$ and $e_2 \Rightarrow v_2'$. Also: logical conjuction of v_1' and v_2' is v_1. Likewise, $e \Rightarrow v_2$ uses And rule and so: $e_1 \Rightarrow v_1''$ and $e_2 \Rightarrow v_2''$ and v_2 is log. conj. of v_1'' and v_2''. By ind hyp. and height of $e_1 \leq k$ and $e_1 \Rightarrow v_1'$ and $e_1 \Rightarrow v_1''$, $v_1' = v_1''$. Likewise, because $e_2 \Rightarrow v_2'$ and $e_2 \Rightarrow v_2''$, we know $v_2' = v_2''$. So $v_1 = v_2$. ✓
- If $e = e_1$ Or e_2, the argument proceeds as in the previous case.

TFb Strengthening Lemma

If $\Gamma\{x:\tau'\} \vdash e:\tau$ and x is not free in e then $\Gamma \vdash e:\tau$.

By induction on height of e.

Base case: height of e is 0. e is one of True, False, an integer, or a variable.

Proceed by case analysis on form of e.

- If $e = \text{True}$ then by Booleen rule, $\Gamma \vdash \text{True} : \text{Bool}$. So it is enough to show $\tau = \text{Bool}$. Because $e = \text{True}$, the proof of $\Gamma\{x:\tau'\} \vdash e:\tau$ must use the Boolean rule. That rule concludes that True has type Bool, so $\tau = \text{Bool}$. ✓