```
let rec sumlist lst =
  match lst with
  | [] -> 0
  | h::t -> h + sumlist t
;;
```

Proof by induction.

Base case: see previous

$P(k)$ implies $P(k+1)$.

If lst has length $k+1$ then h is the first element and t is the rest. So t has length k. By inductive hypothesis: "sumlist t" evaluates to the sum of all elements in t. So "h + sumlist t" evaluates to the sum of all elements in lst. This case is finished.

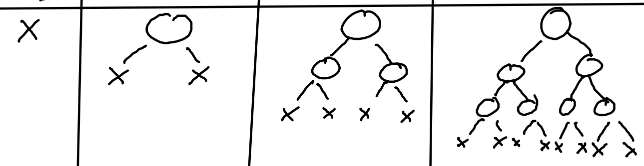
By principle of mathematical induction, $P(n)$ is true for all $n \geq 0$.

$\square$

QED

A perfect binary tree (pbt) is either empty or has two subtrees which are pbts of the same height.

h	-1	0	1	2



Prove that all pbts have $2^{h+1}-1$ nodes where h is the height of the tree.

Proof: by induction on height of pbt.

$P(n) \equiv$ All pbt of height n have $2^{n+1}-1$ nodes.

Prove $P(-1)$: the pbt of height -1 has 0 nodes. $2^{-1+1}-1 = 2^0-1 = 1-1 = 0$.

Prove $P(k)$ implies $P(k+1)$. A pbt of height $k+1$ has a root with two subtrees of the same height. Those subtrees must have height k. By inductive hypothesis, because the subtrees are pbt of height h, they contain $2^{h+1}-1$ nodes each. So this tree has $2\cdot(2^{k+1}-1)+1$ nodes. $2\cdot(2^{k+1}-1)+1 = 2^{k+2}-2+1$

$= 2^{k+2}-1$ nodes.

BOOL

$e ::= v \mid e \text{ And } e \mid e \text{ Or } e \mid \text{Not } e$

$v ::= \text{True} \mid \text{False}$

$$\frac{}{v \Rightarrow v} \qquad \frac{e \Rightarrow \text{True}}{\text{Not } e \Rightarrow \text{False}} \qquad \frac{e \Rightarrow \text{False}}{\text{Not } e \Rightarrow \text{True}} \qquad \frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow \text{True}}{e_1 \text{ And } e_2 \Rightarrow \text{True}} \qquad \frac{\text{False} \in \{v_1, v_2\} \quad e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{False} \quad e_2 \Rightarrow \text{False}}{e_1 \text{ Or } e_2 \Rightarrow \text{False}} \qquad \frac{\text{True} \in \{v_1, v_2\} \quad e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{e_1 \text{ Or } e_2 \Rightarrow \text{True}}$$

"BOOL is normalizing." $\forall e.\ \exists v.\ e \Rightarrow v.$

Proof by strong induction on height of e.

Base case: e has height 0. Therefore, e is a value v. By Value Rule, $v \Rightarrow v$. ✓

Inductive step: e has height k+1. Therefore, e is one of e_1 And e_2, e_1 Or e_2, Not e'.
Proceed by case analysis on form of e.

- $e = \text{Not } e'$. Then e' has height k. By ind hyp., $e' \Rightarrow v'$ for some v'.
  If $v' = \text{True}$, then Not $e' \Rightarrow$ False. If $v' = \text{False}$, Not $e' \Rightarrow$ True.
  So Not $e' \Rightarrow v$ for some v. ✓
- $e = e_1 \text{ And } e_2$. Then e_1 and e_2 have height $\leq k$.