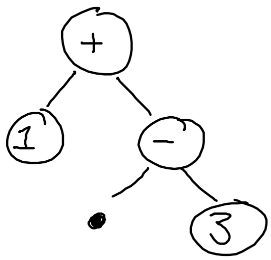
Operational Equivalence

$$4 \cong 1+3$$

A context is an expression with a hole in it.

$$C ::= \bullet \mid \text{Not } C \mid C \text{ Or } e \mid e \text{ Or } C \mid \cdots$$

$1+(\bullet - 3)$



$$(1+(\bullet - 3))[4] = (1+(4-3))$$

Filling a context: we write $C[e]$ to refer to the expression in which e replaces the $\bullet$ in C.

Operational equivalence: we write $e_1 \cong e_2$ to mean

$$\forall C.\ C[e_1] \Rightarrow v_1 \text{ iff } C[e_2] \Rightarrow v_2$$

(for some v_1 and v_2)

Values v_1 and v_2 can be proven to be the same.

$$4 \ncong 5$$

$1 + \bullet$

If $\bullet = 4$ Then 0 Else $1+\text{False}$

Practice w/ contexts: for each of these contexts, find an expression to plug in that produces False. (Then: find an expression that doesn't contain boolean constant.)

(Function a → •) True	False	1 = 8
• 0	(Function a → False)	(Function b → b = 1)
(Function f → Not (f 4)) •	(Function a → True)	(Function a → 1 = 1)

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True $\not\cong$ False	(Function a → If a Then 0 Else 1+True) •
n $\not\cong$ n + 1 - 1	(Function n → Not •) True
(Function a → b) $\not\cong$ (Function a → d)	Let b = 4 In • 3

$\cong$ is an equivalence relation

reflexive	$e \cong e$
symmetric	$e_1 \cong e_2$ iff $e_2 \cong e_1$
transitive	if $e_1 \cong e_2$ and $e_2 \cong e_3$ then $e_1 \cong e_3$

Op eq:

congruent if $e_1 \cong e_2$ then $\forall C.\ C[e_1] \cong C[e_2]$

All divergent expressions are operationally equivalent.

An expression is "used" during evaluation if a subproof shows that it evaluated.

Assume for contradiction that two divergent e_1 and e_2 and some C for which $C[e_1] \Rightarrow v$ and $C[e_2] \not\Rightarrow$.

$C[e_1]$ is closed. Since $C[e_1]$ evaluates, it must not use $\bullet$.

Then $C[e_2]$ does not use $\bullet$. So $C[e_2] \Rightarrow v$.

By contradiction: no such C exists.