$$\boxed{\Gamma \vdash e : \tau \setminus E}$$

$$\frac{v \in \mathbb{Z}}{\Gamma \vdash v : \text{Int} \setminus \emptyset} \qquad \frac{v \in \{\text{True}, \text{False}\}}{\Gamma \vdash v : \text{Bool} \setminus \emptyset} \qquad \frac{(x : \tau) \in \Gamma}{\Gamma \vdash x : \tau \setminus \emptyset}$$

$$\frac{\Gamma\{x : \alpha\} \vdash e : \tau \setminus E \qquad \alpha \text{ is fresh}}{\Gamma \vdash (\text{Function } x \to e) : \alpha \to \tau \setminus E} \qquad \frac{\Gamma \vdash e : \tau \setminus E}{\Gamma \vdash \text{Not } e : \text{Bool} \setminus E \cup \{\tau = \text{Bool}\}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2 \qquad \alpha \text{ is fresh}}{\Gamma \vdash e_1 \; e_2 : \alpha \setminus E_1 \cup E_2 \cup \{\tau_1 = \tau_2 \to \alpha\}}$$

Prove:

$$\dfrac{\dfrac{\dfrac{\dfrac{(b : 'b) \in \{b : 'b\}}{\{b : 'b\} \vdash b : 'b \setminus \emptyset}}{\{b : 'b\} \vdash \text{Not } b : \text{Bool} \setminus \{'b = \text{Bool}\}}}{\emptyset \vdash (\text{Function } b \to \text{Not } b) : 'b \to \text{Bool} \setminus \{'b = \text{Bool}\}} \qquad \dfrac{5 \in \mathbb{Z}}{\emptyset \vdash 5 : \text{Int} \setminus \emptyset}}{\emptyset \vdash (\text{Function } b \to \text{Not } b) \; 5 : 'a \setminus \{'b = \text{Bool}, \; 'b \to \text{Bool} = \text{Int} \to 'a\}}$$

$$'a \setminus \{'b = \text{Bool}, \; 'b \to \text{Bool} = \text{Int} \to 'a\}$$

- $'b = \text{Bool}$
- $'b \to \text{Bool} = \text{Int} \to 'a$
- $'b = \text{Int}$
- $\text{Bool} = 'a$
- $\text{Int} = 'b$
- $\boxed{\text{Int} = \text{Bool}}$

$\text{Int} = \text{Bool}$

$\text{Bool} = \_ \to \_$

$\text{Int} = \_ \to \_$

contradictions are immediate

Closure Rules

1. If $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ then $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$
2. If $\tau_1 = \tau_2$ then $\tau_2 = \tau_1$.
3. If $\tau_1 = \tau_2$ and $\tau_2 = \tau_3$ then $\tau_1 = \tau_3$.

Type Inference

Step 1: derivation

Step 2: deductive closure

Step 3: find contradictions

Step 4: type substitution

Closure Process:

- Apply each closure rule to set every way you can
- Add conclusions to set
- Repeat until set stops changing

$'q \to 'r \setminus \{ 'a = Int,\ 'b = 'a,\ 'c \to 'd = 'b \to Int,\ 'd = Int,\ 'q = 'r \}$ $'q \to 'q$

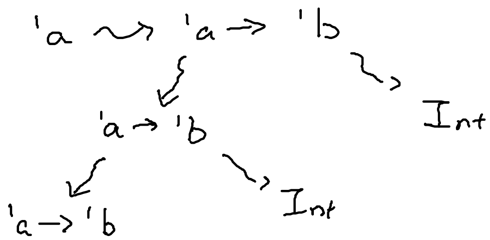
- $'a = Int$
- $'b = 'a$
- $'c \to 'd = 'b \to Int$
- $'d = Int$
- $'b = Int$
- $'c = 'b$
- $'a = 'd$
- $'c = Int$

- $Int = 'a$
- $'a = b$
- $'b \to Int = 'c \to 'd$
- $Int = 'd$
- $Int = 'b$
- $'b = 'c$
- $'d = 'a$
- $Int = 'c$

After deductive closure:
walk through type and replace type vars based on equations

- When a concrete type is available, use it.
- When a type variable is all that is available, use it if it comes earlier in the alphabet.

$'a \setminus \{ 'a = 'a \to 'b,\ Int = 'b \}$



Cycle may occur during type substitution.

You may ignore this for your lab.