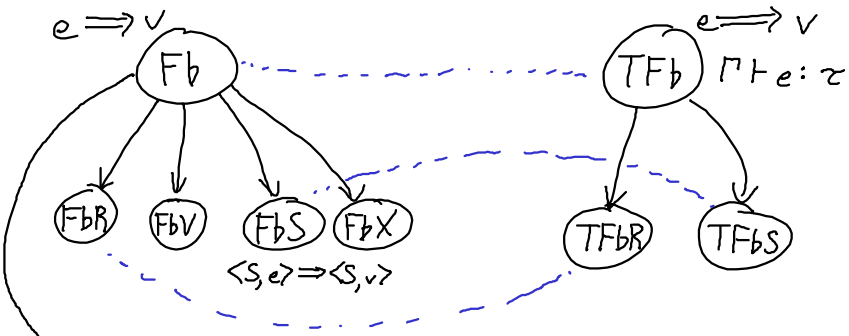
$$\frac{(x:\tau)\in\Gamma}{\Gamma\vdash x:\tau} \qquad \frac{v\in\mathbb{Z}}{\Gamma\vdash v:\mathrm{Int}} \qquad \frac{\Gamma\{x:\tau\}\vdash e:\tau'}{\Gamma\vdash(\mathrm{Function}\ x:\tau\to e):\tau\to\tau'}$$

$$\frac{\Gamma\vdash e_1:\tau_1 \quad\cdots\quad \Gamma\vdash e_n:\tau_n}{\Gamma\vdash\{l_1=e_1;\cdots;l_n=e_n\}:\{l_1:\tau_1;\cdots;l_n:\tau_n\}} \qquad \frac{\Gamma\vdash e:\tau \qquad \tau<:\tau'}{\Gamma\vdash e:\tau'}$$

Prove:

$$\dfrac{\dfrac{4\in\mathbb{Z}}{\{r:\{a:\mathrm{Int}\}\}\vdash 4:\mathrm{Int}}}{\emptyset\vdash(\mathrm{Function}\ r:(\{a:\mathrm{Int}\})\to 4):\{a:\mathrm{Int}\}\to\mathrm{Int}} \qquad \dfrac{\dfrac{\dfrac{R}{\mathrm{Int}<:\mathrm{Int}}}{\{a:\mathrm{Int};b:\mathrm{Int}\}<:\{a:\mathrm{Int}\}}\,\mathrm{SRec} \qquad \dfrac{R}{\mathrm{Int}<:\mathrm{Int}}}{\{a:\mathrm{Int}\}\to\mathrm{Int}\ <:\ \{a:\mathrm{Int};b:\mathrm{Int}\}\to\mathrm{Int}}\,\mathrm{SFun}$$

$$\emptyset\vdash(\mathrm{Function}\ r:(\{a:\mathrm{Int}\})\to 4):\{a:\mathrm{Int};b:\mathrm{Int}\}\to\mathrm{Int}$$



E = "equational constraints"

EFb $\Gamma \vdash e : \tau \setminus E$

Type Inference

$\Gamma ::= \{x : \tau, \ldots\}$

$\tau ::= Int \mid Bool \mid \tau \to \tau \mid \alpha$

$\alpha ::= 'a \mid 'b \mid \ldots$

$E ::= \{\tau = \tau, \ldots\}$

$Int \setminus \emptyset$

$'a \setminus \{'a = Int\}$

$'b \setminus \{Int = 'a,\ 'b = 'a\}$

$'a \setminus \{'a = Int,\ Bool = 'a\}$

$$\Gamma \vdash e : \tau \setminus E$$

Assuming the type environment Γ, e has type τ under the assumptions of the equations E.

1. Infer a type and equation set for expression.
2. Deductively close over equation set.
3. Check for inconsistencies.
4. Type substitution

$\boxed{\Gamma \vdash e : \tau \setminus E}$

Function q → Not q

$$\frac{v \in \mathbb{Z}}{\Gamma \vdash v : \text{Int} \setminus \emptyset} \qquad \frac{(x:\tau) \in \Gamma}{\Gamma \vdash x : \tau \setminus \emptyset}$$

$$\frac{\Gamma\{x:\alpha\} \vdash e : \tau \setminus E \qquad \alpha \text{ is fresh}}{\Gamma \vdash (\text{Function } x \to e) : \alpha \to \tau \setminus E} \qquad \frac{\Gamma \vdash e : \tau \setminus E}{\Gamma \vdash \text{Not } e : \text{Bool} \setminus E \cup \{\tau = \text{Bool}\}}$$

$$\frac{\dfrac{\dfrac{(q:'a) \in \{q:'a\}}{\{q:'a\} \vdash q : 'a \setminus \emptyset}}{\{q:'a\} \vdash \text{Not } q : \text{Bool} \setminus \{'a = \text{Bool}\}}}{\emptyset \vdash (\text{Function } q \to \text{Not } q) : 'a \to \text{Bool} \setminus \{'a = \text{Bool}\}}$$

$'a \to \text{Bool} \setminus \{'a = \text{Bool}\}$

⇓

$\text{Bool} \to \text{Bool}$

$$\frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash e_1 + e_2 : \text{Int} \setminus E_1 \cup E_2 \cup \{\tau_1 = \text{Int}, \tau_2 = \text{Int}\}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2 \qquad \alpha \text{ is fresh}}{\Gamma \vdash e_1\ e_2 : \alpha \setminus E_1 \cup E_2 \cup \{\tau_1 = \tau_2 \to \alpha\}}$$