$$\frac{v \in \{\text{True}, \text{False}\}}{\Gamma \vdash v : \text{Bool}} \qquad \frac{v \in \mathbb{Z}}{\Gamma \vdash v : \text{Int}} \qquad \frac{\Gamma\{x:\tau\} \vdash e : \tau'}{\Gamma \vdash (\text{Function } x:\tau \to e) : \tau \to \tau'}$$

$$\text{Appl} \quad \frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'} \qquad \text{Plus} \quad \frac{\Gamma \vdash e_1 : \text{Int} \qquad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash e_1 + e_2 : \text{Int}}$$

$$\text{Var} \quad \frac{(x:\tau) \in \Gamma}{\Gamma \vdash x : \tau} \qquad \text{Record} \quad \frac{\forall i \in \{1..n\}.\ \Gamma \vdash e_i : \tau_i}{\Gamma \vdash \{l_1 = e_1; \dots; l_n = e_n\} : \{l_1 : \tau_1; \dots; l_n : \tau_n\}}$$

$$\text{Projection} \quad \frac{\Gamma \vdash e : \{l_1 : \tau_1; \dots; l_n : \tau_n\} \quad l = l_k \quad 1 \le k \le n}{\Gamma \vdash e.l : \tau_k}$$

Proof:

$$\dfrac{\dfrac{\dfrac{}{\{r:\{a:\text{Int}\}\} \vdash r : \{a:\text{Int}\}}}{\{r:\{a:\text{Int}\}\} \vdash r.a : \text{Int}}}{\emptyset \vdash (\text{Function } r:\{a:\text{Int}\} \to r.a) : \{a:\text{Int}\} \to \text{Int}} \qquad \dfrac{\dfrac{}{\emptyset \vdash 5 : \text{Int}}}{\emptyset \vdash \{a=5\} : \{a:\text{Int}\}}$$

$$\overline{\emptyset \vdash (\text{Function } r:\{a:\text{Int}\} \to r.a)\ \{a=5\} : \text{Int}}$$

Subtyping

TFbR

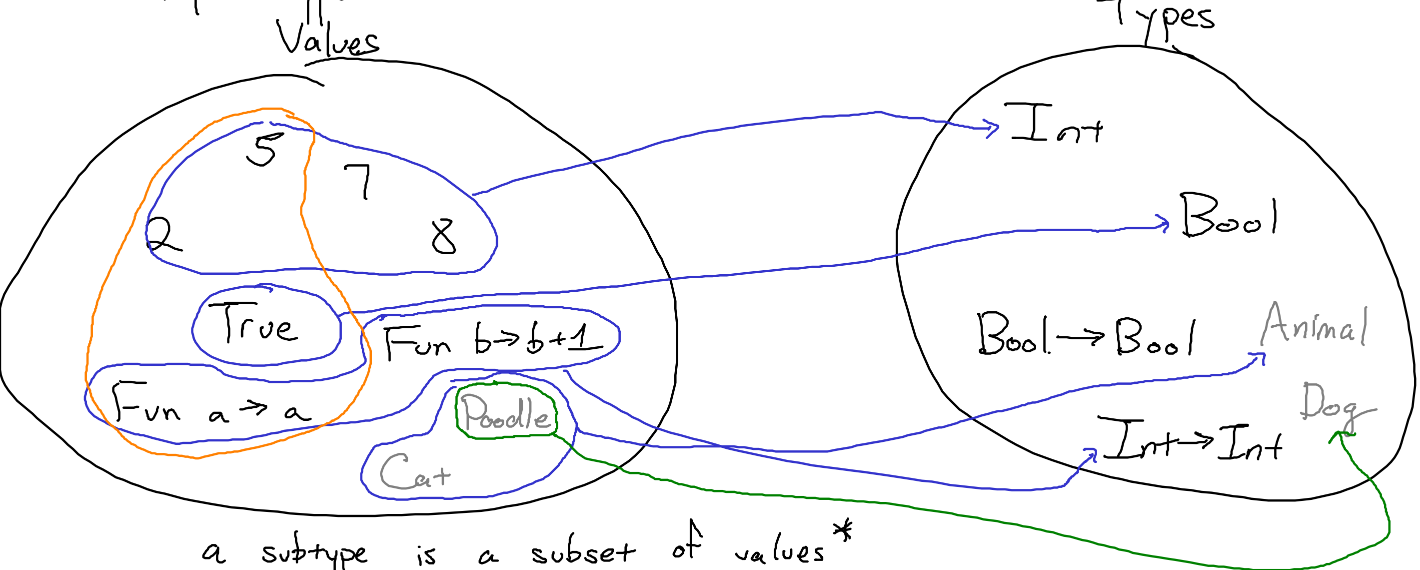
$$\emptyset \nvdash \underbrace{(\text{Function } r:\{a:\text{Int}\} \to r.a)\ \{a=5; b=\text{True}\}}_{\text{want this to typecheck}} \quad \text{type error!}$$

$$(\text{Function } r:\{a:\text{Int}\} \to r.a)\ \{\underset{\text{no } a}{b=\text{True}}\}$$

Subtyping relation

$$\tau <: \tau$$

important types are sets of values* that have something in common



a subtype is a subset of values*

$\boxed{\tau <: \tau}$ "$\tau_1 <: \tau_2$" $\equiv$ every τ_1 is a τ_2

Reflexivity $\dfrac{}{\tau <: \tau}$ Transitivity $\dfrac{\tau_1 <: \tau_2 \qquad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}$

Sub-Record $\dfrac{\tau_1 <: \tau_1' \quad \cdots \quad \tau_n <: \tau_n' \qquad p \leq n}{\{l_1 : \tau_1 ; \ldots ; l_n : \tau_n\} <: \{l_1 : \tau_1' ; \ldots ; l_p : \tau_p'\}}$

$\{a : Int ; b : Bool\}$
$= \{b : Bool ; a : Int\}$

$\{a : Cat ; b : Poodle\} <: \{a : Cat ; b : Dog\}$ depth subtyping: allow subtyping on contained types

$\{a : Cat ; b : Poodle\} <: \{a : Cat\}$

$\{a : Cat ; b : Poodle\} \not<: \{a : Cat ; c : Int\}$

a record subtype has a <u>superset</u> of the labels of its supertype

a type is a promise

Fun a : Int -> a

$$\tau_1 \longrightarrow \tau_2$$

I will accept ANY parameter of τ_1 you give.

I will always give back some τ_2 value a type is a promise

Fun a : Int → a

$$\tau_1 \longrightarrow \tau_2$$

$-$ $\qquad\qquad\qquad\qquad$ $+$

$\forall$ I will accept ANY parameter of τ_1 you give.

I will always give back some τ_2 value $\exists$

$$\frac{\Gamma \vdash e : \tau_1 \qquad \tau_1 <: \tau_2}{\Gamma \vdash e : \tau_2}$$

$$\text{Dog} \to \text{Dog} <: \text{Dog} \to \text{Animal}$$

$$\text{Dog} \to \text{Dog} <: \text{Poodle} \to \text{Dog}$$

Contravariance: this part varies in opposition to the whole

$$\frac{\tau_1' <: \tau_1 \qquad \tau_2 <: \tau_2'}{\tau_1 \to \tau_2 <: \tau_1' \to \tau_2'}$$

Covariance: this part varies in the same way as whole

$$\dfrac{\dfrac{\dfrac{}{\text{Dog} <: \text{Animal}} \qquad \dfrac{}{\text{Poodle} <: \text{Dog}}}{\text{Animal} \to \text{Poodle} <: \text{Dog} \to \text{Dog}} \qquad \dfrac{}{\text{Dog} <: \text{Animal}}}{(\text{Dog} \to \text{Dog}) \to \text{Dog} <: (\text{Animal} \to \text{Poodle}) \to \text{Animal}}$$

$+ \qquad - \qquad +$

$-$

$- \qquad + \qquad +$