Type Systems

type system — system of rules which assign types to expressions

type — a set of values * which have properties we care about

TFb : $\Gamma \vdash e : \tau$

Type Soundness
If $\Gamma \vdash e : \tau$ and $e \Rightarrow v$ then $\Gamma \vdash v : \tau$.

We want this, but it's not guaranteed.

```
if [ ]:
    print ("hi")
```

$$\frac{v \in \not\mathbb{Z}}{\Gamma \vdash v : \text{Bool}}$$

1. We write a proof system.
2. Proof system may or may not align with other proof systems.

$$\frac{v \in \{\text{True}, \text{False}\}}{\Gamma \vdash v : \text{Bool}} \qquad \frac{v \in \mathbb{Z}}{\Gamma \vdash v : \text{Int}} \qquad \frac{\Gamma\{x:\tau\} \vdash e : \tau'}{\Gamma \vdash (\text{Function } x:\tau \to e) : \tau \to \tau'}$$

$$\text{Appl}\ \frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'} \qquad \frac{\Gamma \vdash e_1 : \text{Int} \quad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash e_1 + e_2 : \text{Int}}$$

$$\text{Var}\ \frac{(x:\tau) \in \Gamma}{\Gamma \vdash x : \tau}$$

$$\frac{\dfrac{\dfrac{(a:\text{Int}) \in \{a:\text{Int}\}}{\{a:\text{Int}\} \vdash a : \text{Int}} \qquad \dfrac{4 \in \mathbb{Z}}{\{a:\text{Int}\} \vdash 4 : \text{Int}}}{\{a:\text{Int}\} \vdash a + 4 : \text{Int}}}{\overset{\Gamma}{\emptyset} \vdash (\text{Function } \overset{x}{a}:\overset{\tau}{\text{Int}} \to \overset{e}{a+4}) : \overset{\tau}{\text{Int}} \to \overset{\tau'}{\text{Int}}} \qquad \frac{5 \in \mathbb{Z}}{\emptyset \vdash 5 : \text{Int}}$$

$$\overset{\Gamma}{\emptyset} \vdash \left(\overbrace{(\text{Function } a:\text{Int} \to a+4)}^{e_1}\ \overset{e_2}{5} \right) : \overset{\tau'}{\text{Int}}$$

TFb cannot typecheck ω-combinator or self-passing or Y-combinator

(Function a : (α→β) → a a) (Function a : (α→β) → a a)

$a : \alpha \to \beta \qquad \alpha = \alpha \to \beta$

$a : (\alpha \to \beta) \to \beta$

$a : (\alpha \to \beta) \to \beta \to \beta$

...

Without Let Rec, TFb programs can't recurse

TFbR

$$e ::= \cdots \mid \{l = e; \ldots\} \mid e.l$$

$$v ::= \cdots \mid \{l = v; \ldots\}$$

$$\tau ::= \cdots \mid \{l : \tau; \ldots\}$$

$$\emptyset \vdash \{a = 5; b = 7\} : \{a : Int; b : Int\}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \cdots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash \{l_1 = e_1; \ldots; l_n = e_n\} : \{l_1 : \tau_1; \ldots; l_n : \tau_n\}}$$

$$\frac{\Gamma \vdash e : \{l_1 : \tau_1; \cdots; l_n : \tau_n\} \quad l = l_k \quad k \in \{1..n\}}{\Gamma \vdash e.l : \tau_k}$$

TFbS

$$e ::= \cdots \mid \text{Ref } e \mid !e \mid e := e$$
$$v ::= \cdots \mid c$$
$$\tau ::= \cdots \mid \text{Ref } \tau$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \text{Ref } e : \text{Ref } \tau} \qquad \frac{\Gamma \vdash e : \text{Ref } \tau}{\Gamma \vdash !e : \tau}$$

$$\frac{\Gamma \vdash e_1 : \text{Ref } \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash (e_1 := e_2) : \tau}$$

FbS ~Python

```
Let a = Ref 4 In
a := True
```

TFbS ~C

```
Let a:Ref_ = Ref 4 In
a := True
```