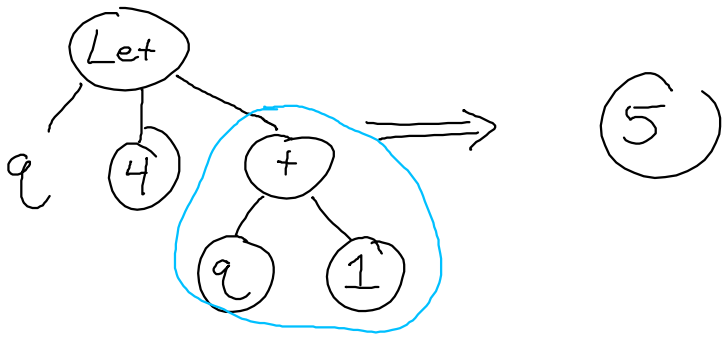
Let q = 4 In q + 1 ⟹ 5

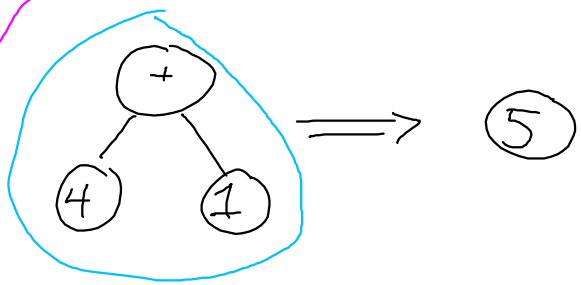
e ::= ... | Let x = e In e
| e = e

definition

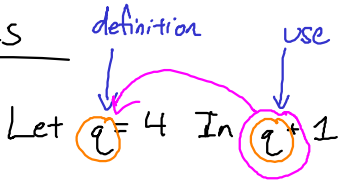
equality



1. Evaluate let expr's first subexpr
2. Replace variable in second subexpr with result of first
3. Evaluate resulting tree



Variables



Let a = 4 In a + b ⟹ free

A variable use is bound to a definition when it refers to that definition.

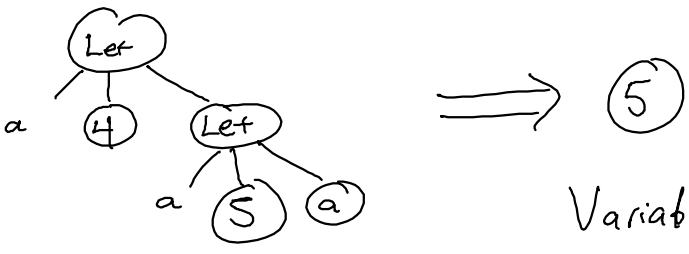
A variable use is free if it refers to no definition.

An open expression is an expression with at least one free variable use.

A closed expression is an expression which is not open.

Open expressions do not evaluate.

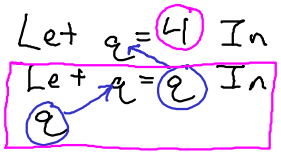
Let a = 4 In Let a = 5 In a




```
class Foo {
public:
  int n = 0;
  void foo() {
    int n = 0;
    n += 4;
```

Variable shadowing: when a variable hides an already-declared variable of the same name

Substitution: $e[v/x]$ $\quad Subst(e, v, x)$



$$True[v/x] = True$$

$$x[v/x] = v$$

$$(e_1 + e_2)[v/x] = (e_1[v/x]) + (e_2[v/x])$$

free for x

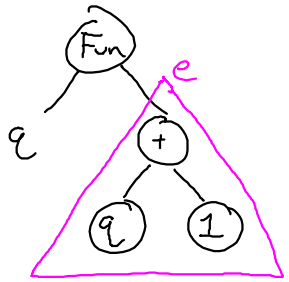
$$(Let\ x = e_1\ In\ e_2)[v/x] = Let\ x = e_1[v/x]\ In\ e_2$$

$$(Let\ x' = e_1\ In\ e_2)[v/x] = Let\ x' = e_1[v/x]\ In\ e_2[v/x] \qquad x \neq x'$$

what we are replacing

where we are replacing it with

$$\frac{e_1 \Rightarrow v_1 \qquad e_2[v_1/x] \Rightarrow v_2}{Let\ x = e_1\ In\ e_2 \Rightarrow v_2}$$



$$\frac{e_1 \Rightarrow Function\ x \to e' \qquad e_2 \Rightarrow v_2 \qquad e'[v_2/x] \Rightarrow v'}{e_1\ e_2 \Rightarrow v'}$$

$$(f\ 4)[(Fun\ n \to n+1)/f] = (Fun\ n \to n+1)\ 4 \Rightarrow 5$$

$$\frac{\dfrac{}{(Fun\ f \to f\ 4) \Rightarrow (Fun\ f \to f\ 4)}\ Val \qquad \dfrac{}{(Fun\ n \to n+1) \Rightarrow (Fun\ n \to n+1)}\ Val \qquad \dfrac{\dfrac{}{(Fun\ n \to n+1) \Rightarrow (Fun\ n \to n+1)}\ Val \qquad \dfrac{}{4 \Rightarrow 4}\ Val \qquad \dfrac{\dfrac{}{1 \Rightarrow 1}\ Val}{4+1 \Rightarrow 5}}{(Fun\ n \to n+1)\ 4 \Rightarrow 5}\ Appl}{(Function\ f \to f\ 4)\ (Function\ n \to n+1) \Rightarrow 5}\ Appl$$

x e' v_2 x e' v_4

$$\frac{}{S} \qquad \frac{S}{\Delta S} \qquad \frac{S}{\frac{\Delta S}{\Delta\Delta S}}$$