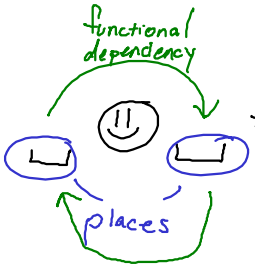
Binary Relations

A binary relation is a set of pairs.

A function is a kind of relation.



$= \{ \vdots, (1,2), (2,3), (5,6), (7,8), \vdots \}$ ← next integer

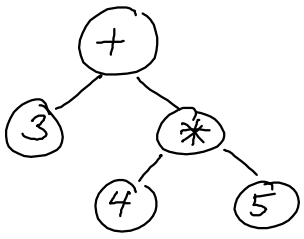
no functional dependencies

$< \; = \{ \vdots, (1,2), (2,3), (3,4), (1,3), (2,5), \vdots \}$

Syntax

```
int foo(int x);
int(foo x;})(
```

3 + 4 * 5



Boolean language

In inference rules: all occurrences of a metavariable must have the same value.

In grammar rules, they don't.

is defined as

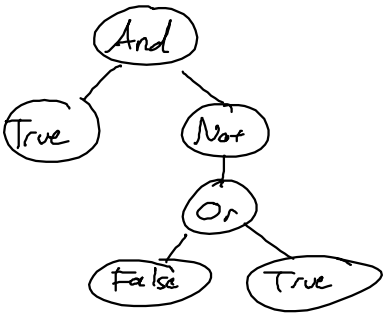
$v ::= \text{True} \mid \text{False}$

EBNF* grammar

$e ::= v \mid \text{Not } e \mid e \text{ And } e \mid e \text{ Or } e$

Not (Not (Not (True)))

True. And (Not (False Or True))




```
type expr =
  | EValue of value
  | ENot of expr
  | EAnd of expr * expr
  | EOr of expr * expr
```

Operational Semantics

a binary relation $e \Rightarrow v$

if b then 4 else 5

b ? 4 : 5

Value Rule $\dfrac{}{v \Rightarrow v}$

$$\frac{e \Rightarrow \text{True}}{\text{Not } e \Rightarrow \text{False}}$$

$$\frac{e \Rightarrow \text{False}}{\text{Not } e \Rightarrow \text{True}}$$

$$\frac{\dfrac{\dfrac{}{\text{True} \Rightarrow \text{True}}}{\text{Not True} \Rightarrow \text{False}}}{\text{Not (Not True)} \Rightarrow \text{True}}}{\text{Not (Not (Not True))} \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow \text{True}}{e_1 \text{ And } e_2 \Rightarrow \text{True}}$$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad \text{False} \in \{v_1, v_2\}}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{False} \quad e_2 \Rightarrow \text{False}}{e_1 \text{ Or } e_2 \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad \text{True} \in \{v_1, v_2\}}{e_1 \text{ Or } e_2 \Rightarrow \text{True}}$$

$$\frac{\vdots \quad \dfrac{\vdots}{e \Rightarrow v_1} \quad \dfrac{\vdots}{\text{Not } e \Rightarrow v_2} \quad \text{True} \in \{v_1, v_2\}}{e \text{ Or Not } e \Rightarrow \text{True}}$$

Admissable — a rule which does not introduce new conclusions

$$\frac{}{e \text{ Or Not } e \Rightarrow \text{True}}$$

Languages / opsems

Normalizing: $\forall e.\ \exists v.\ e \Rightarrow v$

Deterministic: $\forall e, v_1, v_2.$ if $e \Rightarrow v_1$ and $e \Rightarrow v_2$ then $v_1 = v_2$.

Not deterministic

$$\text{Random}\ \frac{v \in \mathbb{N}_4}{\text{Random} \Rightarrow v}$$

Expressions:

Converges: if $\exists v.\ e \Rightarrow v$

Diverges: if $\nexists v.\ e \Rightarrow v$