A Brief Introduction to Logic

set — a collection of elements: all elements are unique and unordered

$\mathbb{Z}$ — set of all integers

$\mathbb{R}$ — set of reals

$\emptyset$ — empty

$\{(x,y) \mid x+y=3\}$ — set of all pairs whose sum of elements is three

$\{4\}$ — set containing 4

$\in$ — "in" $\qquad 5 \in \mathbb{Z}$

$\qquad 1.4 \notin \mathbb{Z}$

$\subseteq$ — subset

If $A \subseteq B$ then every $x \in A$ is also in B.

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proposition — a statement that is either true or false

P is a proposition function

$P(n)$

$$P(n) \equiv \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

today's weather

It is cloudy. FALSE

It is raining. FALSE

It is sunny. TRUE

introduces a metavariable

use of a metavariable

$\forall x \in \mathbb{Z}.\ x^2 \geq 0$ TRUE

$\forall$ — for all

$\exists$ — there exists

For all x, there exist y s.t. $x<y$ $\longleftarrow$ $\forall x.\ \exists y.\ x<y$ TRUE

Every number is smaller than some other number.

There exists a number larger than all numbers. $\longleftarrow$ $\exists y.\ \forall x.\ x<y$ FALSE

If it is Friday and a homework was due yesterday, then I will grade.

$$\text{isFriday}(\text{today}) \underset{\text{and}}{\wedge} \exists \text{homework}.\ (\text{dueYesterday}(\text{homework})) \underset{\text{implies}}{\Longrightarrow} \text{willGrade}(\text{Zach})$$

$\vdash$

Inference rule

$$\frac{\text{premise} \qquad \text{premise}}{\text{conclusion}}$$

$$\frac{\text{isFriday}(\text{today}) \qquad \exists \text{homework}.\ (\text{dueYesterday}(\text{homework}))}{\text{willGrade}(\text{Zach})}$$

$s ::= (\triangle \mid \square \mid \text{ϟ})^*$ "" ϵ — empty string

$1\ \dfrac{\text{ϟ}}{\text{ϟ}\square}$ $2\ \dfrac{\square}{\square\square}$ $3\ \dfrac{\text{ϟ}\square}{\text{ϟ}\triangle}$ $4\ \dfrac{\text{ϟ}\square}{\text{ϟ}\triangle\square}$ $5\ \dfrac{\text{ϟ}\triangle}{\triangle\triangle}$ $6\ \dfrac{}{\text{ϟ}}$

Prove $\triangle\triangle$.

Because 6, ϟ.
By 1 and ϟ, ϟ$\square$.
By 3 and ϟ$\square$, ϟ$\triangle$.
By 5 and ϟ$\triangle$, $\triangle\triangle$.
QED.

$s ::= (\triangle \mid \square \mid \text{ϟ})^*$

$1\ \dfrac{\text{ϟ}}{\text{ϟ}\square\square}$ $2\ \dfrac{s\square}{s\triangle}$ $3\ \dfrac{s\square}{s\triangle\triangle}$ $4\ \dfrac{\text{ϟ}s}{\square s}$ $5\ \dfrac{\square s \triangle}{s}$ $6\ \dfrac{}{\text{ϟ}}$ Inference rules

Prove "".

By 6, ϟ.
By 1 and ϟ, ϟ$\square\square$.
By 3 and ϟ$\square\square$, ϟ$\square\triangle\triangle$. ($s$ = ϟ$\square$)
By 4 and ϟ$\square\triangle\triangle$, $\square\square\triangle\triangle$. ($s = \square\triangle\triangle$)
By 5 and $\square\square\triangle\triangle$, $\square\triangle$. ($s = \square\triangle$)
By 5 and $\square\triangle$, ϵ ($s = \epsilon$)

By 6, ϟ.
By 1 and ϟ, ϟ$\square\square$.
By 4 and ϟ$\square\square$, $\square\square\square$.
By 3 and $\square\square\square$, $\square\square\triangle\triangle$.
⋮

Prove $\square$.

By 6, ϟ.
By 1 and ϟ, ϟ$\square\square$.
By 2 and ϟ$\square\square$, ϟ$\square\triangle$. (s = ϟ$\square$)
By 4 and ϟ$\square\triangle$, $\square\square\triangle$. ($s = \square\triangle$)
By 5 and $\square\square\triangle$, $\square$. ($s = \square$)

$$
\begin{array}{l}
6\ \overline{\quad\text{ϟ}\quad} \\
1\ \overline{\quad\text{ϟ}\square\square\quad} \\
3\ \overline{\quad\text{ϟ}\square\triangle\triangle\quad} \\
4\ \overline{\quad\square\square\triangle\triangle\quad} \\
5\ \overline{\quad\square\triangle\quad} \\
5\ \overline{\quad\epsilon\quad}
\end{array}
$$

Inference proofs

$\wedge$ — and

$\vee$ — or

1 $\dfrac{S}{S\square\square}$ 2 $\dfrac{S_1 \lightning S_2}{S_1 \triangle S_2}$ 3 $\dfrac{S_1 \square\triangle S_2}{S_1 S_2}$ 4 $\dfrac{S_1 \quad \wedge \quad S_2}{S_1 \lightning S_2}$ 5 $\dfrac{}{\lightning}$