

State FbS

Side-effect: anything which <sup>visibly</sup> affects computation other than parameters or return of a function

- \* Mutation (state)
- \* Control flow: exceptions
- \* I/O
- Randomization\*

State FbS

$e ::= \dots \mid \text{Ref } e \mid e := e \mid !e$   
 $v ::= \dots \mid c$   
 $c ::= (\text{infinite set of cell identifiers})$  *no concrete syntax*  
 $S ::= \{c \mapsto v, \dots\}$  (such that  $c$  is unique)

$$\frac{e \Rightarrow v \quad \text{pick a new } c \text{ associate } v \text{ with } c}{\text{Ref } e \Rightarrow c}$$

$\{ \#0 \mapsto \text{True} \}$

Let  $a = \text{Ref } 4+4$  In  
 $\vdots$

Evaluation of  
 $\text{Fb}, \text{FbP}, \text{FbR}, \text{FbV}$   
 as relation  
 $e \Rightarrow v$

FbS evaluation relation:  $\langle S, e \rangle \Rightarrow \langle S, v \rangle$

$\mapsto$   
 \mapsto

$$\text{Ref } \frac{\langle S, e \rangle \Rightarrow \langle S', v \rangle \quad c \notin S' \quad S'' = S' \{c \mapsto v\}}{\langle S, \text{Ref } e \rangle \Rightarrow \langle S'', c \rangle}$$

Let  $f_{00} =$   
 $\text{cell} := 5$

$$\text{Assign } \frac{\langle S, e_1 \rangle \Rightarrow \langle S', c \rangle \quad \langle S', e_2 \rangle \Rightarrow \langle S'', v_2 \rangle \quad S''' = S'' \{c \mapsto v_2\}}{\langle S, e_1 := e_2 \rangle \Rightarrow \langle S''', v_2 \rangle}$$

In  
 $\dots$

$\text{OC}_{\text{cell}} := \text{unit}$   
 $c :$

$$\text{Deref } \frac{\langle S, e \rangle \Rightarrow \langle S', c \rangle \quad (c \mapsto v) \in S'}{\langle S, !e \rangle \Rightarrow \langle S', v \rangle}$$

In C language:

$$\text{Value } \frac{}{\langle S, v \rangle \Rightarrow \langle S, v \rangle}$$

$w = 5;$   
 $a = b = c = 0;$

$$\text{Plus } \frac{\langle S_1, e_1 \rangle \Rightarrow \langle S_2, v_1 \rangle \quad \langle S_2, e_2 \rangle \Rightarrow \langle S_3, v_2 \rangle \quad v \text{ is the sum of } v_1 \text{ and } v_2}{\langle S_1, e_1 + e_2 \rangle \Rightarrow \langle S_3, v \rangle}$$

Let  $a = \text{Ref } 4$  In  
 $(!a) + (a := !a + 1)$

$$\langle \emptyset, 4 \rangle \Rightarrow \langle \emptyset, 4 \rangle$$

$$\langle \emptyset, \text{Ref } 4 \rangle \Rightarrow \langle \{ *1 \mapsto 4 \}, *1 \rangle$$

$$\langle \emptyset, \text{Let } a = \text{Ref } 4 \text{ In } (!a) + (a := !a + 1) \rangle \Rightarrow$$

$$e ::= \dots \mid e; e$$

Sequence

$$\frac{\langle S_1, e_1 \rangle \Rightarrow \langle S_2, v_1 \rangle \quad \langle S_2, e_2 \rangle \Rightarrow \langle S_3, v_2 \rangle}{\langle S_1, e_1; e_2 \rangle \Rightarrow \langle S_3, v_2 \rangle}$$