A Brief Introduction to Logic

A proposition is a statement which can be evaluated for truth.

Ex. It is cloudy. TRUE
I am flying. FALSE

$P(n)$ is "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$"

It is cloudy and raining. FALSE

$\text{cloudy}(\text{today}) \wedge \text{raining}(\text{today})$ — $\wedge$ "and"; $\vee$ "or"

$\forall$ "for all"
$\exists$ "there exists"

$\forall \text{day} \in \text{days}.\ \text{cloudy}(\text{day})$ FALSE.

(one of the days; element of / set of days)

TRUE	$\forall x.\ \exists y.\ x > y$	for every number there is another number which is less than it
FALSE	$\exists x.\ \forall y.\ x > y$	there is a number bigger than every other number

If it is the weekend and I have no work then I will sleep in.

$\Rightarrow$
$\vdash$

$\text{weekend}(\text{today}) \wedge \text{work}(\text{me}) = \emptyset \Rightarrow \text{sleepingIn}(\text{me})$

We will not be using this for implication

Inference Rule

format $\frac{\text{premise}}{\text{conclusion}}$

$$\frac{\text{weekend}(\text{today}) \qquad \text{work}(\text{me}) = \emptyset}{\text{sleepingIn}(\text{me})}$$

Inference Proofs

$S ::= (\Delta \mid \lightning \mid \square)^*$ "" $\in$ (empty string)

Δ

$\Delta\square$

$\square\lightning\lightning$

1 $\frac{\lightning}{\lightning\square}$ 2 $\frac{\square}{\square\square}$ 3 $\frac{\lightning\square}{\lightning\Delta}$ 4 $\frac{\lightning\square}{\lightning\Delta\square}$ 5 $\frac{\lightning\Delta}{\Delta\Delta}$ 6 $\frac{}{\lightning}$

Prove $\Delta\Delta$

$\lightning$ thus $\lightning\square$ thus $\lightning\Delta$ thus $\Delta\Delta$

6 $\frac{}{\lightning}$
1 $\frac{\lightning}{\lightning\square}$
3 $\frac{\lightning\square}{\lightning\Delta}$
5 $\frac{\lightning\Delta}{\Delta\Delta}$

$S ::= (\Delta \mid \lightning \mid \square)^*$

1 $\frac{\lightning}{\lightning\square\square}$ 2 $\frac{S\square}{S\Delta}$ 3 $\frac{S\square}{S\Delta\Delta}$ 4 $\frac{\lightning S}{\square S}$ 5 $\frac{\square S\Delta}{S}$ 6 $\frac{}{\lightning}$

Prove "". Prove $\square$.

6 $\frac{}{\lightning}$
1 $\frac{\lightning}{\lightning\square\square}$
s="□□" 4 $\frac{}{\square\square\square}$
s="□□" 3 $\frac{}{\square\square\Delta\Delta}$
s="□Δ" 5 $\frac{}{\square\Delta}$
s=ϵ 5 $\frac{}{\epsilon}$

s=ϵ 6 $\frac{}{\lightning}$ 4 $\frac{}{\square}$

6 $\frac{}{\lightning}$
1 $\frac{}{\lightning\square\square}$
4 $\frac{}{\square\square\square}$
2 $\frac{}{\square\square\Delta}$
5 $\frac{}{\square}$

1 $\frac{S}{S\square\square}$ 2 $\frac{S_1 \lightning S_2}{S_1\Delta\lightning S_2}$ 3 $\frac{S_1\square\Delta S_2}{S_1 S_2}$ 4 $\frac{S_1 \quad S_2}{S_1 \lightning S_2}$ 5 $\frac{}{\lightning}$

Prove: $\Delta\Delta\lightning\lightning\lightning\square\square$.

5 $\frac{}{\lightning}$
$S_1=\epsilon$ $S_2=\epsilon$ 2 $\frac{}{\Delta\lightning}$
$S_1=\Delta$ $S_2=\epsilon$ 2 $\frac{}{\Delta\Delta\lightning}$
s=⚡ 1 $\frac{}{\lightning\square\square}$
4 $\frac{}{\Delta\Delta\lightning\lightning\lightning\square\square}$

S_1 S_2