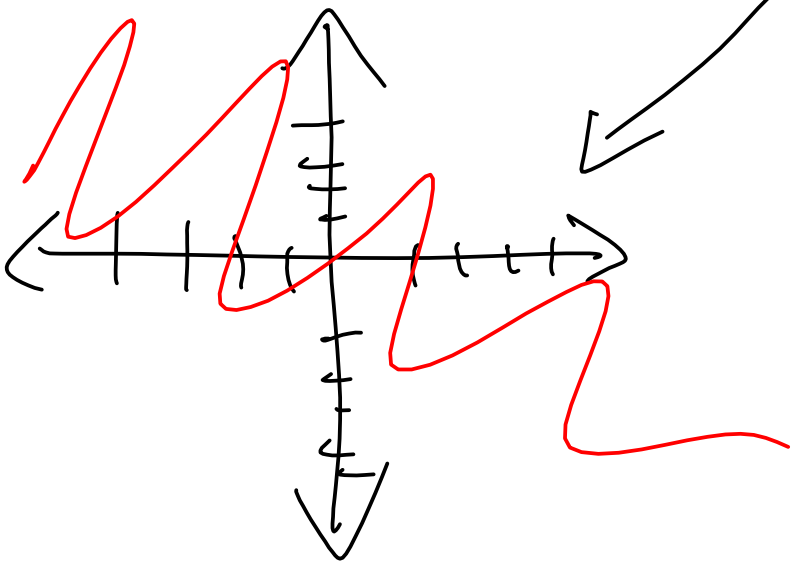
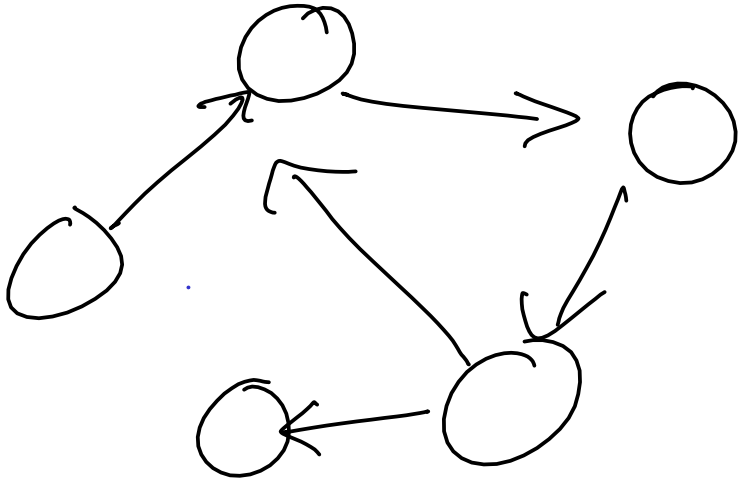
Graphs



NOT THIS ONE



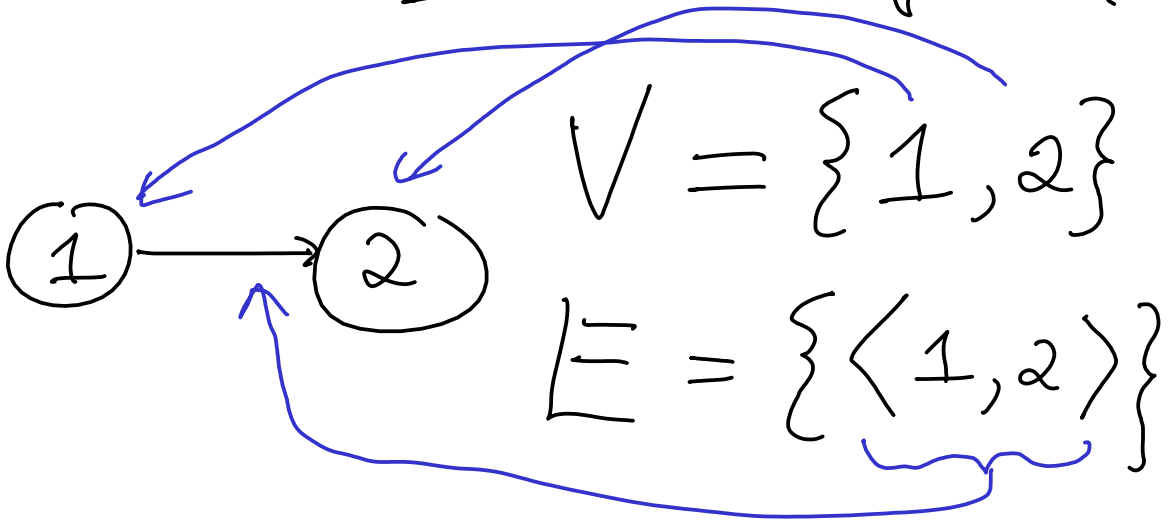
Graphs

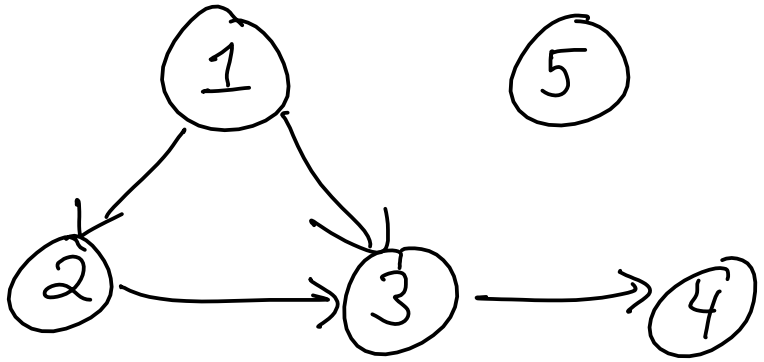
connections / relationships / interactions

A graph is a pair $\langle V, E \rangle$ of sets

V (vertex set — anything) and E (edge set)

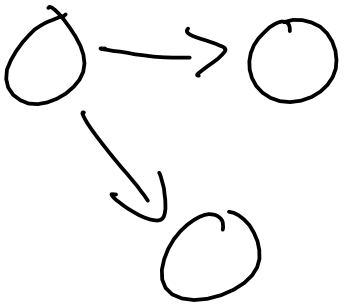
An edge has two vertices: source and target (destination)





$V = \{1, 2, 3, 4, 5\}$

$E = \{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 2, 3\rangle, \langle 1, 3\rangle\}$

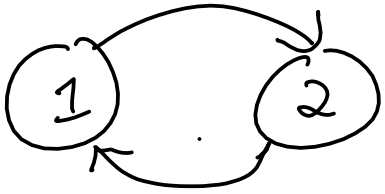


Tree is a kind of graph

In a tree, no loops

In a tree, each vertex is the target of at most one edge

Trees are connected

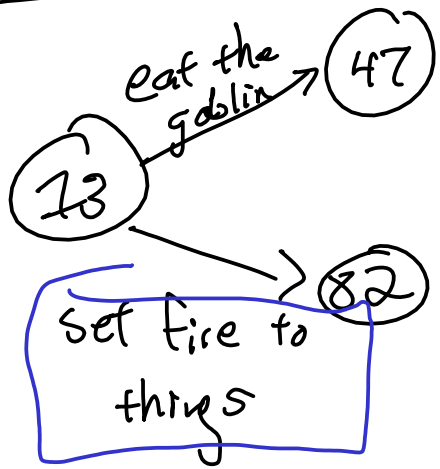


$V = \{1, 2\}$

$E = \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$

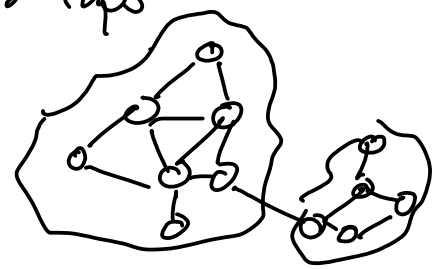
Graphs

Choose Your Own Adventure

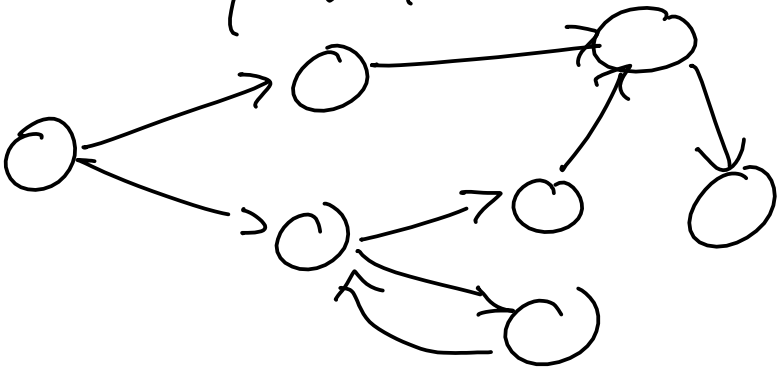


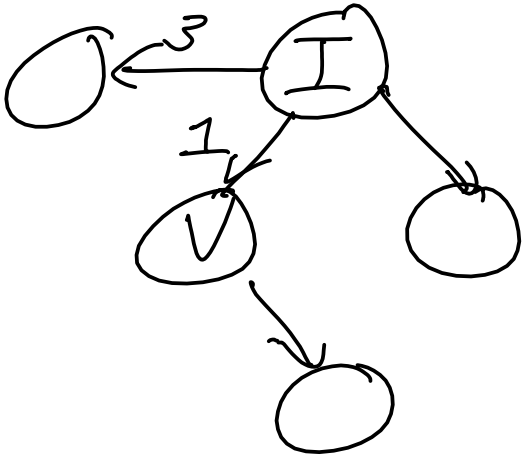
neural networks

Maps

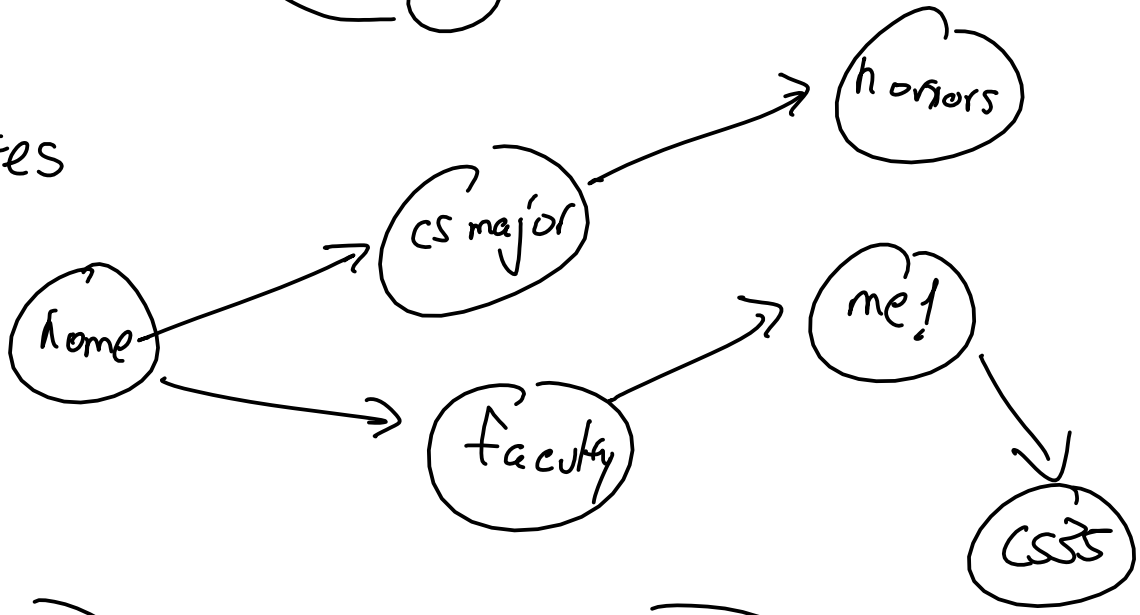


dependency graph





Websites



social networks

Edges also have label and weight

Some data

Comparable (usually) numeric

Vertex

Edge (source vertex, target vertex, label, weight)

Maps

Vertex = GPS location

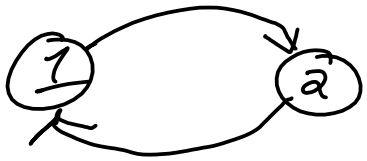
Weight = distance / speed (time) — both

Label = direction / name of route

Directed

not that ⟶ if $\langle v_1, v_2 \rangle \in E$ then $\langle v_2, v_1 \rangle \in E$

Undirected



Weighted

not that ⟶

Unweighted

all weights same

weights don't matter

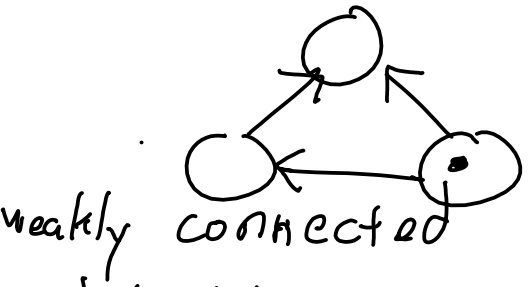
Connected Graph

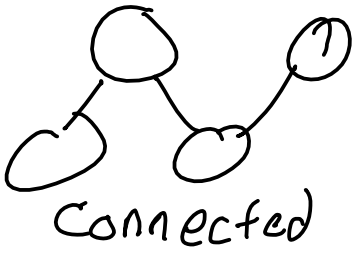
Path: list of edges from E where each target is followed by same source

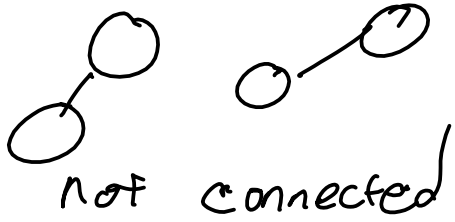
$[\langle 1,2\rangle, \langle 2,4\rangle, \langle 4,3\rangle]$ path

$[\langle 1,2\rangle, \langle 3,5\rangle, \langle 5,2\rangle]$ not path

Connected graph: all vertices have a path to all other vertices



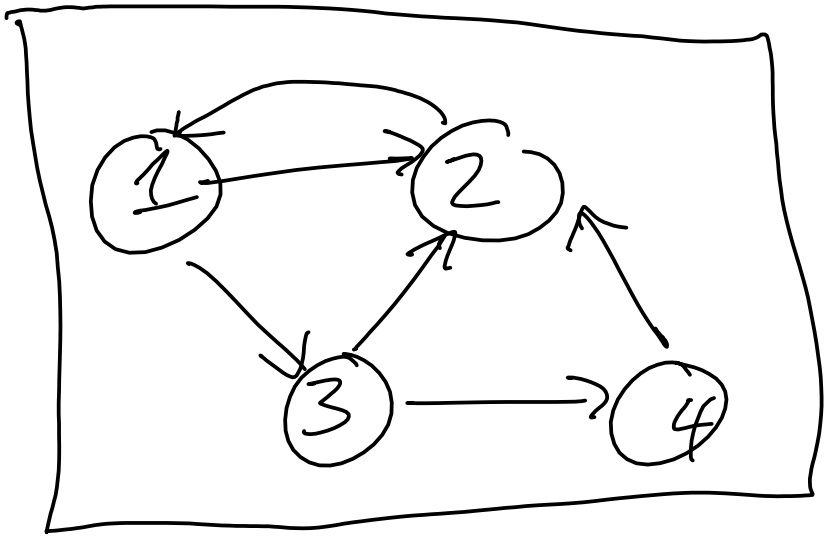




Weakly connected graph: a graph that would be connected if it were undirected

Each vertex has "degree" : in-degree + out-degree

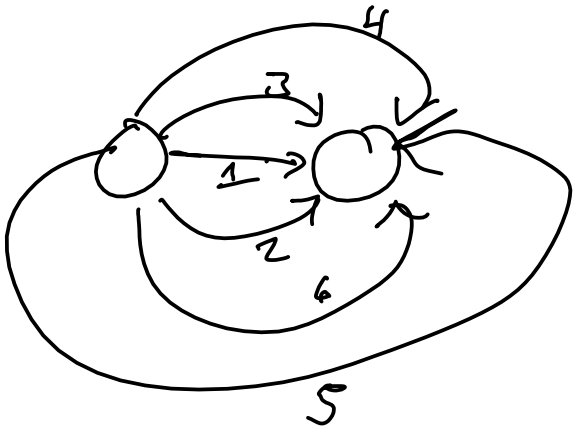
- in-degree : # incoming edges (where this vertex is target)
- out-degree : # outgoing edges

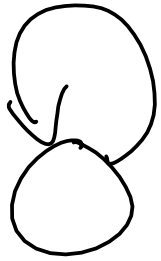


simple graphs

simple if

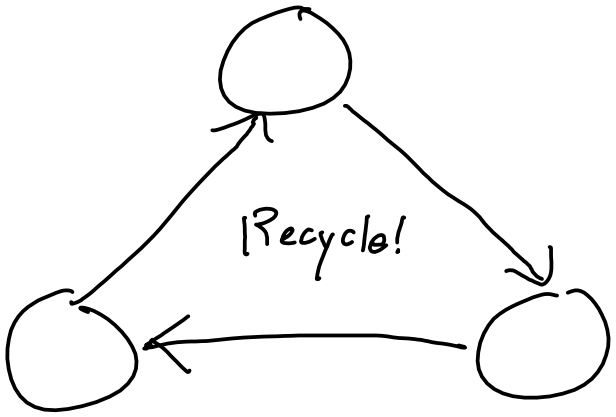
- each source & target has at most 1 edge
- no edge has source = target





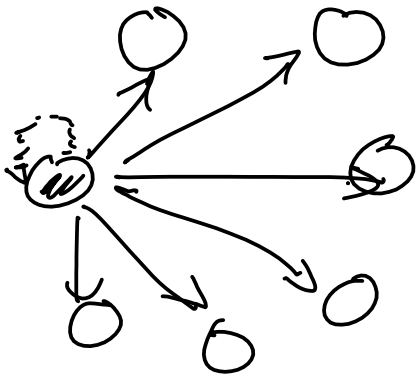
vertices in a graph $|V|$

edges in graph $|E|$

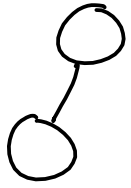


given $|V|$, what is max $|E|$?

$$|V|(|V|-1) = \boxed{|V|^2 - |V|}$$



if undirected graph as only counting each pair once



1 edge

$$\frac{|V|(|V|-1)}{2}$$

Given $|E|$, what is sum of degree of all vertices?

$2|E|$

each edge gives 1 in-degree
1 out degree