Principal Components Analysis (PCA) example

Step 1: Get the data. In this small example we will have $n = 6$ data points and $p = 2$ features. In reality we would have many more of each, and sometimes $p >> n$. The data matrix with $n$ rows and $p$ columns is called $X_{\text{orig}}$:

$$X_{\text{orig}} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Step 2: Subtract off the column-wise mean from each column (feature) to obtain $X$ (fill in above). The mean of column $f$ is:

$$\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f_i$$

Step 3: Compute the covariance of each pair of features in $X$ to obtain the $p \times p$ covariance matrix $A$. The covariance of feature $f$ with feature $g$ is:

$$\text{cov}(f, g) = \frac{1}{n-1} \sum_{i=1}^{n} (f_i - \bar{f})(g_i - \bar{g})$$

Note that in our case, we have set all the means to be 0. Also note that variance is a special case when $f = g$:

$$\text{cov}(f, f) = \text{var}(f) = \frac{1}{n-1} \sum_{i=1}^{n} (f_i - \bar{f})^2$$

Fill in $A$ below:

$$A = \begin{bmatrix} \end{bmatrix}$$

Step 4: Compute the eigenvalues ($\lambda_1, \lambda_2$ for $p = 2$) and eigenvectors ($\vec{v}_1, \vec{v}_2$) of $A$. The eigenvectors (sorted by eigenvalue) will become the directions of our principal components (i.e. new coordinate system). We want our eigenvectors and eigenvalues to satisfy:

$$A\vec{v} = \lambda \vec{v} \Rightarrow \det(A - \lambda I) = 0$$
Step 5: Transform the data $X$ using the eigenvector matrix $W$ (one eigenvector on each column, sorted by eigenvalue). The number of eigenvectors we use corresponds to the number of dimensions we retain. Say we want to retain $r$ dimensions, then we would obtain the transformed data $T_r = XW_r$. $T_r$ will be an $n \times r$ matrix. In our case, use $r = 2$ and compute $T_r$.

$$T_2 = XW_2 = \begin{pmatrix} dlfdkjd \end{pmatrix} \begin{pmatrix} dlfdkjd \end{pmatrix} = \begin{pmatrix} dlfdkjd \end{pmatrix}$$

Step 6: Finally, plot the transformed data $T_r$ with principal component 1 (PC1) on the $x$-axis and PC2 on the $y$-axis. We can plot further PCs on different coordinate systems.