Bayesian Probability

(find and work with a partner)

Clinical Trials Example. Say the probability of a disease (D) in the general population is 1 in 100, i.e. \( P(D) = \frac{1}{100} \). This is our prior probability of the disease (i.e. without any data).

Furthermore, say we have test for this disease with 90% accuracy. We will call the results of the test positive ("pos") and negative ("neg"). 90% accuracy means that \( P(\text{pos}|D) = \frac{9}{10} \), and \( P(\text{neg}|H) = \frac{9}{10} \), where H means healthy.

What we actually want to know is: what is the probability of having the disease, given a positive test?

1. Apply Bayes rule to \( P(D|\text{pos}) \). Recall that Bayes rule says:
   \[
P(A|B) = \frac{P(A)P(B|A)}{P(B)}.
   \]

2. We can often write the denominator as the sum of \( P(a, B) \), for all options \( a \in \text{vals}(A) \):
   \[
P(A|B) = \frac{P(A)P(B|A)}{\sum_{a \in \text{vals}(A)} P(a, B)}.
   \]
   Use this idea to expand the denominator in the clinical trials example and compute a numerical value for \( P(D|\text{pos}) \).