In-lab notes: April 5

- Posterior decoding
- Posterior mean
- Working in log-space
Lab A
Yesterday
\[ P(x, \ldots, x_i, z_i = k) \]
end in state \( k \)
obs through \( x_i \)

\[ f_i(k) = P(x, \ldots, x_i, z_i = k) \]

termination

\[ P(\vec{x}) = \sum_{k} P(\vec{x}, z_k = k) \]

\[ P(\vec{x}) = \sum_{k} f(\vec{z}_k) \]

last state

likelihood

\[ \log P(\vec{x}) \leftarrow \log \text{-likelihood} \]
\[
P(z_i = k \mid x) = \frac{f_k(i) b_k(i)}{P(x)}
\]

\[
\hat{z}_i = \text{arg max}_k P(z_i = k \mid x)
\]
Posterior mean

\[ j_c = \sum P(z_i=k|\theta)g(z_i) \]

\[ g(z) = 4.54 \]

\[ z^* = \{0, 2, 5, 1, 0, 2\} \]

\[ L = 3 \]

Order

- Viterbi
  - check path & graph

- Forward
  - check log-likelihood

- Backward
  - posterior → decoding → mean
\[ \log \left( V_k(i) \right) = \log(e_k(x_i)) + \max_j \left( V(i-1)_j \cdot a_k \right) \]

\[ \tilde{V}_k(i) = \log(e_k(x_i)) + \max_j \left( \tilde{V}_k(i-1) + \log(a_k) \right) \]

log 0 is bad!

\[ \log(p+q+r) \]
\[ f_k(i) = e_k(x_i) \sum_{l} f_{k(l-1)} a(k_l) \]

\[ \hat{p} = \log p \]
\[ q = \log q \]

\[ \log(p + q) = \log(e^{\hat{p}} + e^{\hat{q}}) \]

\[ = \log(e^{\hat{p}} (1 + e^{-\hat{q}})) \]

\[ = \hat{p} + \log(1 + e^{\hat{q} - \hat{p}}) \]
Lab B
Yesterday

\[ P_c(k) = P(x_1, \ldots, x_t, z_i = k) \]

obs through \( x_i \)

end in state \( k \)

termination

\[ P(\tilde{x}) =\sum_k P(\tilde{x}, z_c = k) \]

\[ P(\tilde{x}) = \sum_k f(k) \]

likelihood of data

\[ \log (P(\tilde{x})) \leftarrow \log \text{likelihood} \]

given \( \tilde{x} \)

posterior probability

\[ P(z_i | \tilde{x}) \]
\[ P(\tau_i = k | \mathbf{x}) = \frac{f_k(i) b_k(i)}{P(\mathbf{x})} \]

\[ \tau_i = \arg\max_k P(\tau_i = k | \mathbf{x}) \]

Posterior decoding

\[ \hat{y} = \left\{ 0, 2, 3, 1, 0, 2 \right\} \]

Column max

\[ L = [1, 2, 0, 3, 2, 4] \]
\[ g(k) = \text{time of state } k \]
\[ g(2) = 4.54 \]

\[ \overline{g_i} = \sum_{k=1}^{L} P(x_i = k | x) g(k) \]

- Viterbi
  - check path + graph
    - forward
    - backward
  - posterior
    - decoding
    - mean
log-space

\[ \log(V(i)) = e_k(x_i) \cdot \max_e \{ V_e(i-1) a e_k \} \]

\[ V_e(i) = \log(e_k(x_i)) + \max_e \{ \widetilde{V}_e(i-1) + \log(a e_k) \} \]

- Increasing!
- \( \log x \)
- \( \log(0) \) is bad!

- \( \log(p + (q + r)) \)

- \( P(S') = 0 \)
- \( P(S' \leq h \leq S'' \text{(odd)}) \geq 0 \)

- \( S \geq 3 \)
- \( S = 1238 \ldots \)
\[ \log(f_k(i)) = e^z_i \sum_l f_l(i-1) \alpha e^k \]

become 0!

\[ \tilde{p} = \log p, \quad \tilde{q} = \log q_0 \]

\[ \log(p + q) \]

\[ = \log(e^{\tilde{p}} + e^{\tilde{q}}) = \log(e^{\tilde{p}}(1 + e^{\tilde{q} - \tilde{p}})) \]

\[ = \tilde{p} + \log(1 + e^{\tilde{q} - \tilde{p}}) \]