Review problems

1. **Lighting:** You are given a unit sphere centered at the origin and light source specified with the following code:

```javascript
var pointLight = new THREE.PointLight("white", 2, 30); // color, intensity, distance light travels
pointLight.position.set( 2, 2, 0 );
```

What is the unit normal vector ($\vec{n}$) at the point $\vec{p}= (\sqrt{2}, \sqrt{2}, 0)$ on the sphere? What is the unit direction of the light ray ($\vec{l}$) pointing towards this point? Compute the dot product of these two unit vectors. What does this tell us about the color of sphere at this point?

2. **Projection:** *(adapted from the Fall 2015 final exam)* You are given the following 8 vertices of a cube in world space, a camera at the origin, and a viewport at $z = -1$.

(a) Fill in the table below with the 2D viewport coordinates for each type of projection. Assume the viewport is large enough that no points will be clipped out.

<table>
<thead>
<tr>
<th>world coordinates</th>
<th>orthographic projection</th>
<th>perspective projection</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1,-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1,-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,3,-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3,-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,1,-4)</td>
<td></td>
<td></td>
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<tr>
<td>(3,1,-4)</td>
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<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Draw what the “viewer” would see in each case (only one quadrant of the viewport is shown).

Orthographic

Perspective
3. Name the algorithm based on the pseudocode below:

   for all p in pixels:
       create a ray r from camera to p
       for all o in objects of the world:
           calculate intersection of o with r
           keep if closest
       color p based on material of o & angle of surface to light

4. Ray tracing: (adapted from the Fall 2015 final exam) You are trying to figure out whether a circular mirror on a wall in your scene is visible from a certain pixel. The direction of the ray and the point on the viewport are, respectively:

   \[ \vec{R}_d = \left( \frac{3}{5}, 0, -\frac{4}{5} \right), \quad \text{and} \quad \vec{p}_v = (-1, 2, -1) \quad (z = -1 \text{ included for clarity}). \]

   (a) The camera is 5 units away from the point \( \vec{p}_v \). Where is the camera located (find \( \vec{R}_0 \))?

   (b) The circular mirror is located on the wall represented by the plane \( x = 5 \). What are the coordinates of \( \vec{p}_w \), the point where this ray intersects the wall? How far away is \( \vec{p}_w \) from the camera?

   (c) The circular mirror has center point \( \vec{c} = (5, 5, -7) \) and radius \( r = 3 \). Does this ray intersect the mirror? Justify your answer (it might be helpful to draw a picture of the wall).

   (d) Create a general algorithm for determining whether a ray intersects a circular object lying on a given plane. You don’t need to use code or pseudocode, just a general description of how to find the equivalent of the point on the plane \( \vec{p}_w \), and then an inequality in terms of \( \vec{p}_w, \vec{c} \), and \( r \).