announcements

- project lab
  - overall goal
  - replication section(s)
- project proposal due 1:00 am Wednesday, 10/28
reminder: reaction notes

• “3 things”
• something neat
• unclear/more info
intro to game theory

model of strategic interactions between multiple players
utility matrix
captures preferences of players
project dilemma

- # players
  - 2 member project group
- actions/strategies
  - goof off
  - work hard
- preferences
  - best case “taking advantage of”
  - worst case being “taken advantage of”
  - prefer both work hard to both goof off
preferences as utilities

• prefer both work hard to both goof off
• worst case being “taken advantage of”
• best case “taking advantage of”

• \( u_1(H, H) > u_1(G, G) \)
• worst case: \( u_1(H, G) \)
• best case: \( u_1(G, H) \)

\( G \): goof off \hspace{1cm} \( H \): work hard
**project dilemma**

\[ u_1(G, H) > u_1(H, H) > u_1(G, G) > u_1(H, G) \]

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<th>Group Member 2</th>
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<tr>
<td><strong>H</strong></td>
<td>(2,2)</td>
<td>(0,3)</td>
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<tr>
<td><strong>G</strong></td>
<td>(3,0)</td>
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**G**: goof off  
**H**: work hard
arms race

• # players
  • 2 countries

• actions/strategies
  • build nuclear bomb
  • don’t build bomb

• preferences
  • worst case being disarmed when other country is armed
  • best case being armed when other country is disarmed
  • prefer both disarmed to both armed
compare the utility matrices

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**G**: goof off  
**H**: work hard  
**D**: disarmed  
**A**: armed
friends with differing tastes

• 2 friends
• options:
  • chopin without piano
  • steve jobs
friends preferences

• worst case
  being alone
• friend 1 prefers
  the movie
• friend 2 prefers
  the play
define the utility function

\[ u_1(M, M) > u_1(P, P) > u_1(P, M) = u_1(M, P) \]
\[ u_2(P, P) > u_2(M, M) > u_2(P, M) = u_2(M, P) \]

\textbf{M}: movie \quad \textbf{P}: play
game in normal form

\[ u_1(M, M) > u_1(P, P) > u_1(P, M) = u_1(M, P) \]
\[ u_2(P, P) > u_2(M, M) > u_2(P, M) = u_2(M, P) \]

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**M**: movie  
**P**: play
compare the utility matrices

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**D:** disarmed  
**A:** armed  

**M:** movie  
**P:** play
pure conflict: matching pennies

• 2 players
• choose:
  • heads
  • tails
matching pennies utility matrix

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<tr>
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<td>(-1, 1)</td>
<td>(1, -1)</td>
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H: heads
T: tails
pure conflict: penalty kicks

• 2 players
• choose:
  • left
  • right
so far:
capture preferences of players

up next:
identifying **likely outcomes** of player interactions
## cs97 group roulette

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**D**: disarm  
**A**: arm
Finding a Nash Equilibrium

A strategy profile \( s=(D, D) \) is a Nash Equilibrium (NE) iff

\[
\forall r_1 \in \{D, A\} \quad u_1(D, D) \geq u_1(r_1, D)
\]

and

\[
\forall r_2 \in \{D, A\} \quad u_2(D, D) \geq u_2(D, r_2)
\]
Finding a Nash Equilibrium

a strategy profile \( s=(D, A) \) is a NE iff

\[
\forall r_1 \in \{D, A\} \quad u_1(D, A) \geq u_1(r_1, A)
\]

and

\[
\forall r_2 \in \{D, A\} \quad u_2(D, A) \geq u_2(D, r_2)
\]
Finding a Nash Equilibrium

a strategy profile $s = (A, D)$ is a NE iff

$$\forall r_1 \in \{D, A\} \ u_1(A, D) \geq u_1(r_1, D)$$

and

$$\forall r_2 \in \{H, G\} \ u_2(A, D) \geq u_2(A, r_2)$$
Finding a Nash Equilibrium

A strategy profile $s = (A, A)$ is a NE iff

$$\forall r_1 \in \{D, A\} \quad u_1(A, A) \geq u_1(r_1, A)$$

and

$$\forall r_2 \in \{D, A\} \quad u_2(A, A) \geq u_2(A, r_2)$$
An unfortunate Nash Equilibrium

A strategy profile \( s=(s_1, s_2) \) is a {\textbf{NE}} iff

\[
\forall r_1 \in S_1 \ u_1(s_1, s_2) \geq u_1(r_1, s_2)
\]

and

\[
\forall r_2 \in S_2 \ u_2(s_1, s_2) \geq u_2(s_1, r_2)
\]
More generally...

for an $n$-player game

a strategy profile $s = (s_1, s_2, \ldots, s_n)$ is a NE iff

$\forall$ player $i$

$\forall r_i \in S_i \quad u_i(s_1, s_2, \ldots, s_{i-1}, r_i, s_{i+1}, \ldots, s_n) \geq u_i(s_1, s_2, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_n)$
More generally...

for an \( n \)-player game

a strategy profile \( s = (s_1, s_2, \ldots, s_n) \) is

a \textbf{NE} iff

\[
\forall \text{ player } i
\]

\[
\forall r_i \in S_i \quad u_i(s) \geq u_i(r_i, s_{-i})
\]