CS97 Section 01

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group meetings

project proposals
reaction notes

- Key research question
- Key contributions
  - significance
  - results
- optional: limitations/future work
- 2 discussion questions
reaction notes: example

Corporate Prediction Markets

• research question
  • how well do corporate prediction markets perform? how “efficient” are they at information aggregation?

• Key contributions
  • compare to “public” markets (thinness, weak incentives, biases)
  • use actual data from deployed markets: google, ford, “firm x”.
  • compare favorably to “expert” predictions, get better with time, optimism bias, bias away from naive priors
  • issues: limited data, scrapped market

• 2 discussion questions
  • is the success due to selection bias?
  • why defunct? how can these be encouraged?
market mechanism design

- (proper) scoring rules and MSR market makers

- information theoretic properties of log score

- equivalent formulation in terms of cost function
Scoring Rules

Uncertain future event
Forecaster predicts $p$
The true state of the world $x$ is revealed
We pay forecaster according to score:

$S(p;x)$

(examples we have seen: quadratic, log)
Quadratic Scoring Rule is Proper

\[ x \in \{0,1\} \]

Belief \( p \):

\[ E_p[S(p, x)] \geq E_p[S(p', x)] \]

\[ \iff E_p[- (x - p)^2] - E_p[- (x - p')^2] \geq 0 \]

\[ \iff E_p[- (x - p)^2 + (x - p')^2] \geq 0 \]

\[ \iff E_p[(x - p' - x + p)(x - p' + x - p)] \geq 0 \]

\[ \iff E_p[(p - p')(2x - p' - p)] \geq 0 \]

\[ \iff (p - p')(2E_p[x] - p' - p) \geq 0 \]

\[ \iff (p - p')(p - p') \geq 0 \]

\[ S(p; x) = - (x - p)^2 \]

\[ (a - b)(a + b) = a^2 - b^2 \]

\[ E[cx] = c \ E[x] \]

\[ E[x_1 + x_2] = E[x_1] + E[x_2] \]
properties of the logarithmic scoring rule
Log Scoring Rule

Forecaster predicts $p$
The true state of the world $x \in \{0, 1\}$ is revealed
We pay forecaster according to the log score:

$$S(p; x) = \begin{cases} 
\log p & \text{if } x = 1 \\
\log 1-p & \text{if } x = 0 
\end{cases}$$
entropy

\[ H(p) = - \sum_{i=1}^{n} p_i \log p_i \]

entropy is non-negative

\[ H(p) = - p \log p - (1-p) \log (1-p) \]
consider a risk-neutral score maximizing trader
What is the **expected** log score of a trader who believes event A happens \( \text{wp 0.5} \)?

What is the **wc** log score of a trader who believes event A happens \( \text{wp 0.5} \)?

What is the **best case** log score of a trader who believes event A happens \( \text{wp 0.5} \)?
What is the **expected** log score of a trader who believes event A happens wp 1?

What is the **wc** log score of a trader who believes event A happens wp 1?

What is the **best case** log score of a trader who believes event A happens wp 1?
expected log score

more certain, greater expected score

Note: be clear about the difference between the \textit{wc} score and \textbf{expected} score
alternate formulation of the LMSR
LMSR Cost Function

\[ C(q) = \log \sum_{i=1}^{n} e^{q_i} \]

Already purchased: \( q_1 \) shares \( q_2 \) shares

Wants to purchase: \( r_1 \) shares \( r_2 \) shares

Cost of purchase:

\[ C \left( \left( \frac{q_1}{q_2} \right) + \left( \frac{r_1}{r_2} \right) \right) - C \left( \left( \frac{q_1}{q_2} \right) \right) = \log \frac{e^{q_1+r_1} + e^{q_2+r_2}}{e^{q_1} + e^{q_2}} \]
Cost Function Prediction Market

Already purchased:

- $q_1$ shares
- $q_2$ shares

Instantaneous prices:

- $p_1 = \frac{\partial C(q)}{\partial q_1} = \frac{e^{q_1}}{e^{q_1} + e^{q_2}}$
- $p_2 = \frac{\partial C(q)}{\partial q_2} = \frac{e^{q_2}}{e^{q_1} + e^{q_2}}$

“predictions”
from the perspective of a risk neutral trader

$\text{cost function formulation} \equiv \text{sequential scoring rule formulation}$