L(M) = \{ w\#w \mid w \in \{0,1\}^* \}
Back to the Beginning

Idea: Use a *special symbol* and unique property of being at left-most cell
Shifting to the right

\[
\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 1 & \ddots & \\
0 & 1 & 1 & 0 & 0 & 1 & \ddots \\
\end{array}
\]
Design a **decider** for the **element distinctness** problem

\[ E = \{ \#x_1\#x_2\#\ldots\#x_n \mid \text{each } x_i \in \{0,1\}^* \text{ and } x_i \neq x_j \text{ for each } i \neq j \} \]

**idea:** *marking* symbols
Variants of TMs

1. TMs with stay-put
2. Multitape TMs
3. Nondeterministic TMs (NTMs)
TM with stay put

∀ z ∈ Σ
Multitape TM

\[(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\]

\[\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k\]
Every multitape TM has an equivalent single tape TM

Idea: The tape contents and the head positions of M can be represented on the single tape of S and correspondingly updated; S accepts iff M accepts.
starting configuration
intermediate configuration

M

S

# a a b # b a # a b a a # ...
1. **remember** current (marked) symbols
2. **replace** marked symbols as indicated by M’s transition function
3. **move** marked symbols
   a. possibly **shift** tape contents right
The total number of pages on this exam is indicated in the footer.
Please check that you have a complete exam.

Name: ______________________________
username: __________________________

Instructions:

• Please write your name and username in the space provided.

• You have the entirety of lab time to finish this exam.

• This is a closed book, closed notes exam. However, you may have access to a “reminder” sheet restricted to one side of a US Letter size paper (8.5” × 11”).

• Please turn off all cell phones and electronic devices. Headphones, computers, wireless devices, notebooks, textbooks, and other external objects that are or appear to be capable of communication, computation, or storage must be stowed out of reach during the exam.

• Please write your solutions in the space provided. If you run out of room for an answer, continue on the back of the page.

• Write legibly! In extreme cases, lack of readability may result in a points penalty.

• Unless otherwise instructed, show enough work to justify your answer.

• When giving proofs, you may use any results shown in class, in the text, in the practice problems or homework without proving those results.

• You should attempt all problems. Partial solutions will receive partial credit.

• Good luck!
(a) Prove something
(b) Prove something else
Problem types:
proofs, constructions, simulations
Chapter 0:
Concepts and Example problems

sets, subsets, power sets

function f: $A \rightarrow B$

proof structure

determining set membership and cardinality
Chapters 1 & 2: Concepts and Example problems

designing a DFA/NFA/RE/PDA/CFG

determining language recognized

RE $\leftrightarrow$ NFA $\leftrightarrow$ DFA

Pumping lemma

Closure

Simulation
Designing a (simple) TM

determining language recognized

configurations

decidable/recognizable languages