Variants of TMs

1. TMs with stay-put
2. Multitape TMs
3. Nondeterministic TMs (NTMs)
4. TM with output (enumerators)
TM with stay put

∀ z ∈ Σ
Multitape TM

\((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\)

\[\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k\]
Every multitape TM has an equivalent single tape TM

Idea: The tape contents and the head positions of M can be represented on the single tape of S and correspondingly updated; S accepts iff M accepts.
starting configuration

M

\[
\begin{array}{cccccccccccccccc}
\text{a} & \text{a} & \text{b} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{...} \\
\text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{...} \\
\text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{...} \\
\end{array}
\]

S

\[
\begin{array}{cccccccccccccccc}
\# & \text{a} & \text{a} & \text{b} & \# & \text{u} & \# & \text{u} & \# & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{u} & \text{...} \\
\end{array}
\]
intermediate configuration

M

S

# a a b # b a # a b a a # u ...

a a b u u u u u u u u u u ...

b a u u u u u u u u u u ...

a b a a u u u u u u u u u ...

…
1. remember current (marked) symbols
2. replace marked symbols as indicated by M’s transition function
3. move marked symbols
   a. possibly shift tape contents right
Breadth First Search

1. remove first node from queue
2. add children to queue
Breadth First Search

order of traversal:

\[ \varepsilon, 1, 2, 3, 11, 12, 13, 21, 22, 23, 31, 32, 33 \]
Depth First Search

1. pop first node from stack
2. push children to stack

stack:
Depth First Search

order of traversal:

$\epsilon$ 3 33 32 31 2 23 22 21 13 11 12 11
Nondeterministic TM

\[(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\]

\[\delta : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R\})\]
Nondeterministic TM

each node corresponds to a configuration
Nondeterministic TM: Accept
Nondeterministic TM: nonterminating branch
Theorems

Obs 1:
Every multitape TM has an equivalent single tape TM

Every nondeterministic TM has an equivalent deterministic TM
Obs 2: max number of children = 3
Given: NTM $N$

Construct: Multitape Deterministic TM $D$ that simulates every branch of $N$
Idea: traverse using BFS