$L(M) = \{ \$ \ 0^n = 0^n \mid n \geq 0 \}$
Configuration of a TM

- current state
- current tape contents
- current head location

state control
$q_3$

read head

a a b b a U U U U U U U ...

tape

a $q_3$ a b b a
The set of strings accepted by a TM $M$ is called its **language** $L(M)$. $L(M)$ is said to be **recognized** by $M$.

A language is called **Turing recognizable** iff some TM recognizes it.
(Turing) Decidable

A TM called a **decider** iff it always halts on all inputs.

A language is called **(Turing) decidable** iff some decider recognizes (decides) it.
Is the above TM a
A. recognizer
B. decider
C. undetermined
Is the above TM a
A. recognizer
B. decider
C. undetermined
\[ L(M) = \{ w\#w \mid w \in \{0,1\}^* \} \]
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Check if the first (alphabet) symbol to the right of # is the same as a
If so replace with x, else reject
Go left beyond # until you see x
Move R
Replace $a$ with $x$ and remember $a$
Go $R$ past $#$ and $x$

$$L(M) = \{w\#w \mid w \in \{0,1\}^*\}$$
\[ L(M) = \{ w\#w \mid w \in \{0,1\}^* \} \]

Check if the first (alphabet) symbol to the right of \# is the same as \texttt{a}.
If so replace with \texttt{x}, else \texttt{reject}.
Go left beyond \# until you see \texttt{x}.
Move \texttt{R}.
Replace a with x and remember a
Go R past # and x
Check if the first (alphabet) symbol to the right of # is the same as a
If so replace with x, else reject

\[ L(M) = \{ w#w \mid w \in \{0,1\}^* \} \]
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Check if the first (alphabet) symbol to the right of \# is the same as \texttt{a}.
If so replace with \texttt{x}, else \texttt{reject}.
Go left beyond \# until you see \texttt{x}.
Move \texttt{R}.
If the first symbol you see is \# 
Go R past all x. If you see a ⊔ accept

accept!

\[ L(M) = \{ w\#w \mid w \in \{0,1\}^* \} \]
$L(M) = \{ w\#w \mid w \in \{0,1\}^* \}$
What is the 3rd in the sequence of configurations $M$ enters on input: 010#010

A. $xxx\#xxx \ q_{\text{accept}}$
B. $010q_1\#010$
C. $x10q_1\#010$
D. $01q_10\#010$
E. $x1q_10\#010$
input: 010#010
Shifting to the right

1 1 0 0 1

0 1 1 0 0 1

...
Shifting to the right

Idea:
• read and remember the current symbol
• while current symbol \( \neq \sqcup \)
  • move right
  • read and remember the current symbol
  • write previously remembered symbol
Shifting to the right

\[ \Sigma = \{0, 1\} \]
Idea: Use a special symbol and unique property of being at left-most cell
Back to the Beginning

Read and remember current symbol
Replace with a special symbol
Try to move L
Read current symbol
If not special symbol
   Read and remember current symbol
   Replace with special symbol
Back to the Beginning

Read current symbol
If not special symbol
  Read and remember current symbol
  Replace with special symbol
Move R
  Replace with previously remembered symbol and come back
Try to move L
Back to the Beginning

Try to move **L**
Read current symbol
If special symbol
**success!**
replace x with previously remembered symbol
Design a **decider** for

\[ L(M) = \{0^{2^n} \mid n \geq 0\} \]
Design a **decider** for the **element distinctness** problem

\[ E = \{ \#x_1\#x_2\#...\#x_n \mid \text{each } x_i \in \{0,1\}^* \text{ and } x_i \neq x_j \text{ for each } i \neq j \} \]

**Idea:** *marking* symbols