Pumping Lemma:
Context Free Languages

If A is a context free language
then there is a pumping length p st
if $s \in A$ with $|s| \geq p$
then we can write $s = uvxyz$ so that

- $\forall i \geq 0 \; uv^ixy^iz \in A$
- $|vy| > 0$
- $|vxy| \leq p$

To prove $\{a^n b^n c^n \mid n \geq 0\}$ is *not* context free using the Pumping Lemma

- Suppose $\{a^n b^n c^n \mid n \geq 0\}$ is context free.
- Let $s = a^p b^p c^p$
- The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
  - $|vy| > 0$
  - $uvvxyyz \in A$

<u>Case 1:</u> both v and y contain at most one type of symbol

<u>Case 2:</u> either v or y contain more than one type of symbol

To prove $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is *not* context free using the Pumping Lemma

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- Let $s = a^p b^p c^p$
- *some* split $s = uvxyz$
- $|vy| > 0$

<u>Case 1:</u> both v and y contain at most one type of symbol

(i) a's not included $\Rightarrow uxz \notin A$

(ii) b's not included

  (i) a's included $\Rightarrow uvvxyyz \notin A$

  (ii) c's included $\Rightarrow uxz \notin A$

(iii) c's not included

  (i) a's included $\Rightarrow uvvxyyz \notin A$

  (ii) b's included $\Rightarrow uvvxyyz \notin A$

To prove $\{ww \mid w \in \{0,1\}^*\}$ is *not* context free using the Pumping Lemma

1. Suppose $\{ww \mid w \in \{0,1\}^*\}$ is context free
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$. Let $\mathbf{s = 0^p10^p1}$
4. The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
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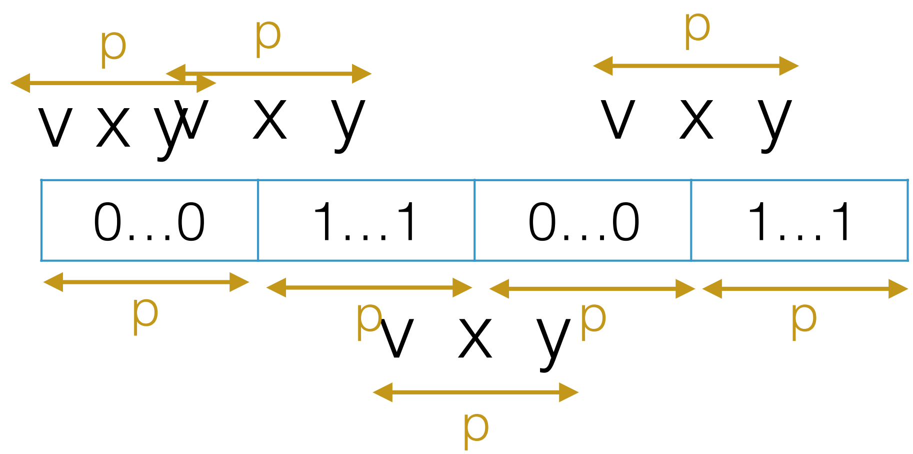
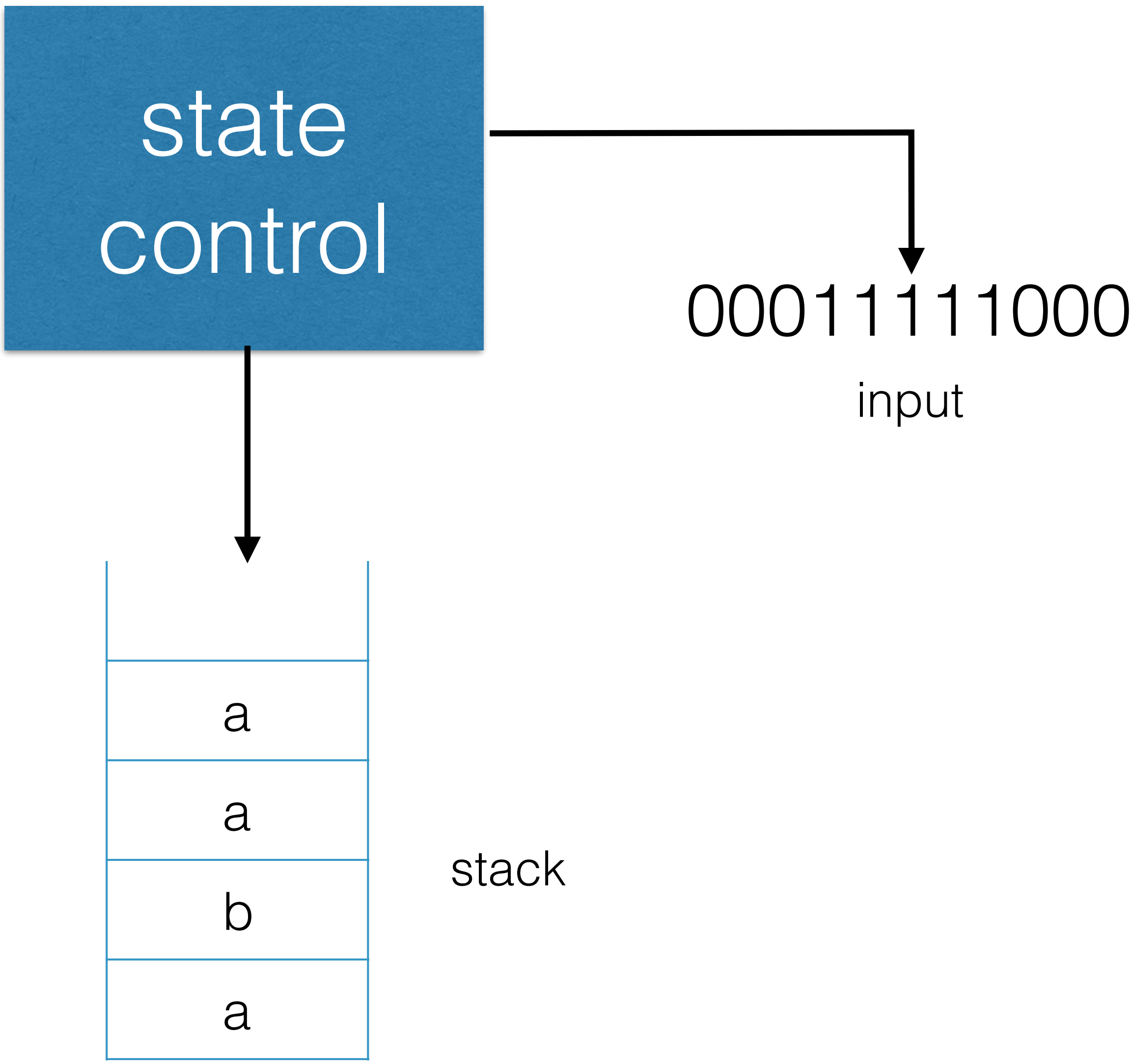
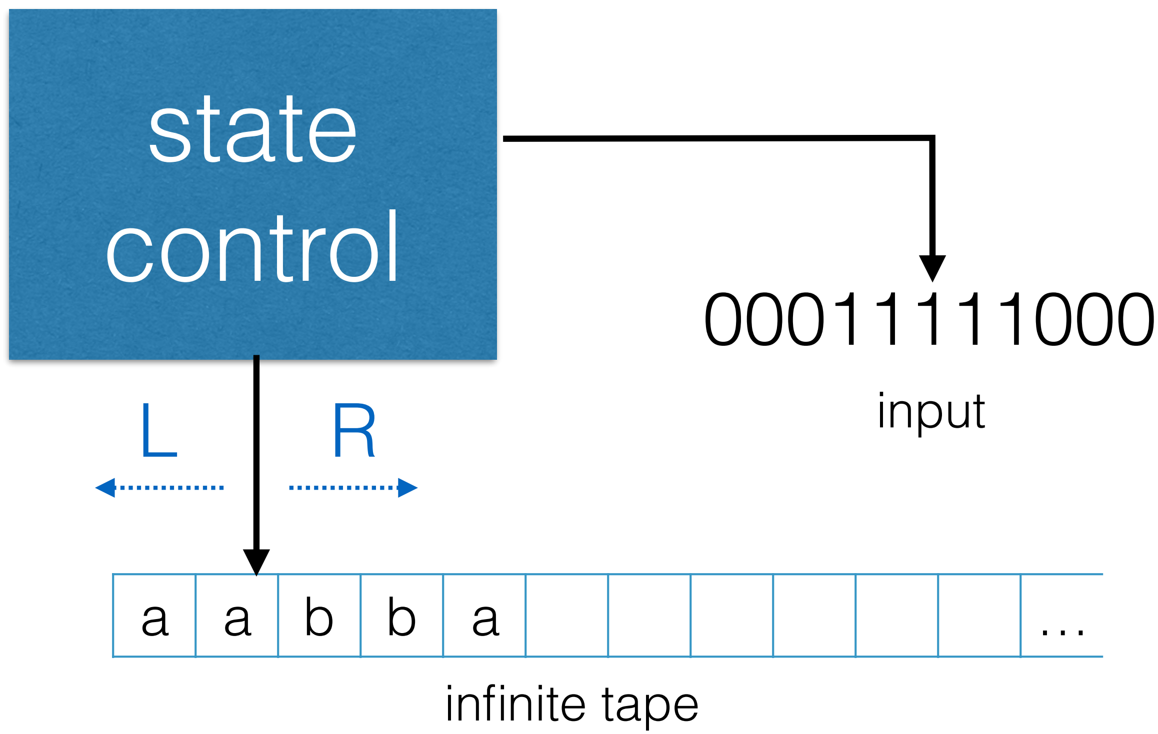


$uvvxyyz \notin A$

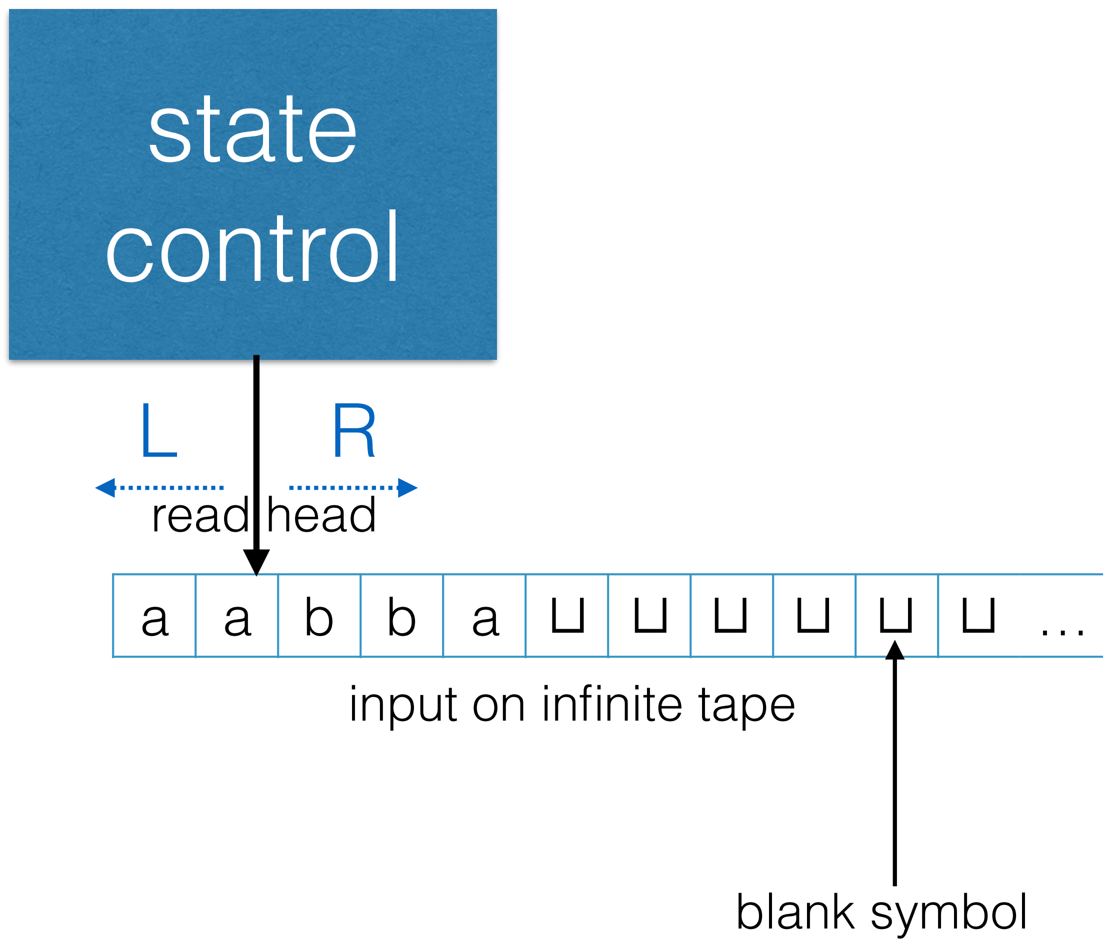
Pushdown Automata (PDA)



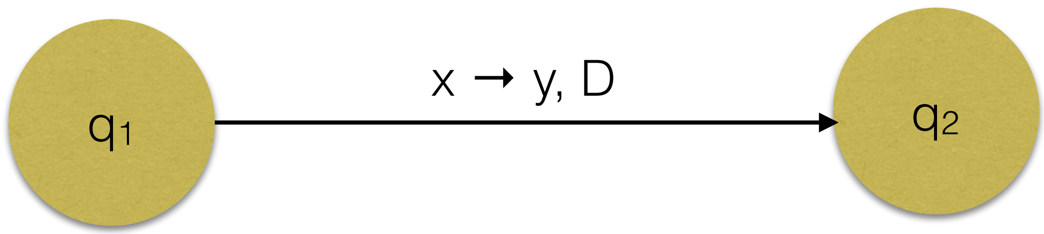
Turing Machines



Turing Machines



Turing Machines (TM)

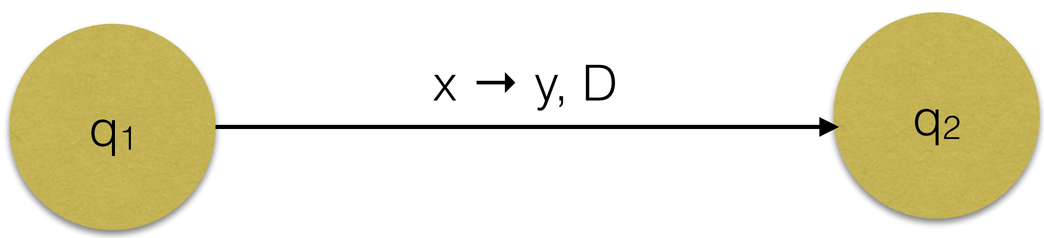


If the current tape symbol is x then

- transition from q_1 to q_2
- replace x with y
- advance tape head in direction $D \in \{L, R\}$

Turing Machines (TM)

special case 1

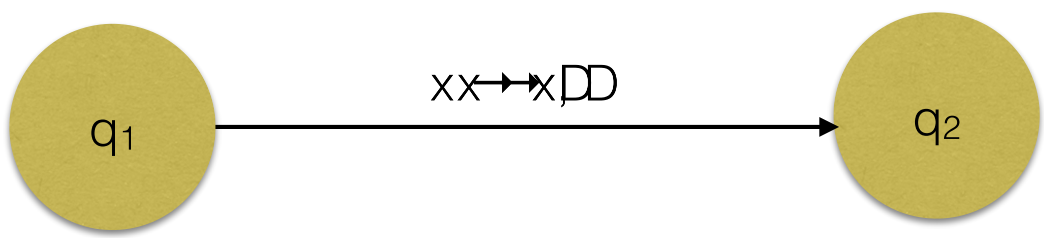


If the current tape symbol is x then

- transition from q_1 to q_2
- advance tape head in direction $D \in \{L, R\}$
    - if the tape head is on first cell, and D = L, *stay put*

Turing Machines (TM)

special case 2



If the current tape symbol is x then

- transition from q_1 to q_2
- advance tape head in direction $D \in \{L, R\}$

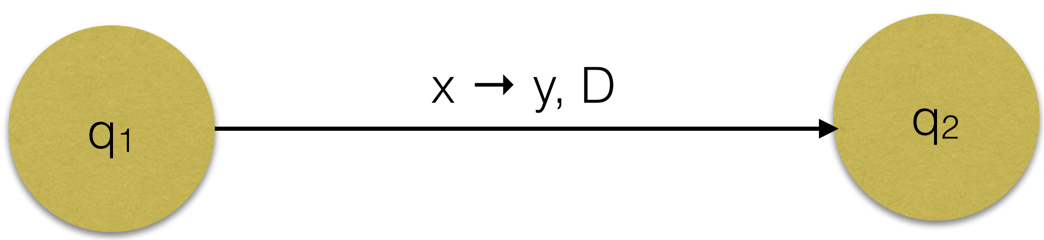
Turing Machines (TM)

does *not* contain ⊔

halting states

$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$

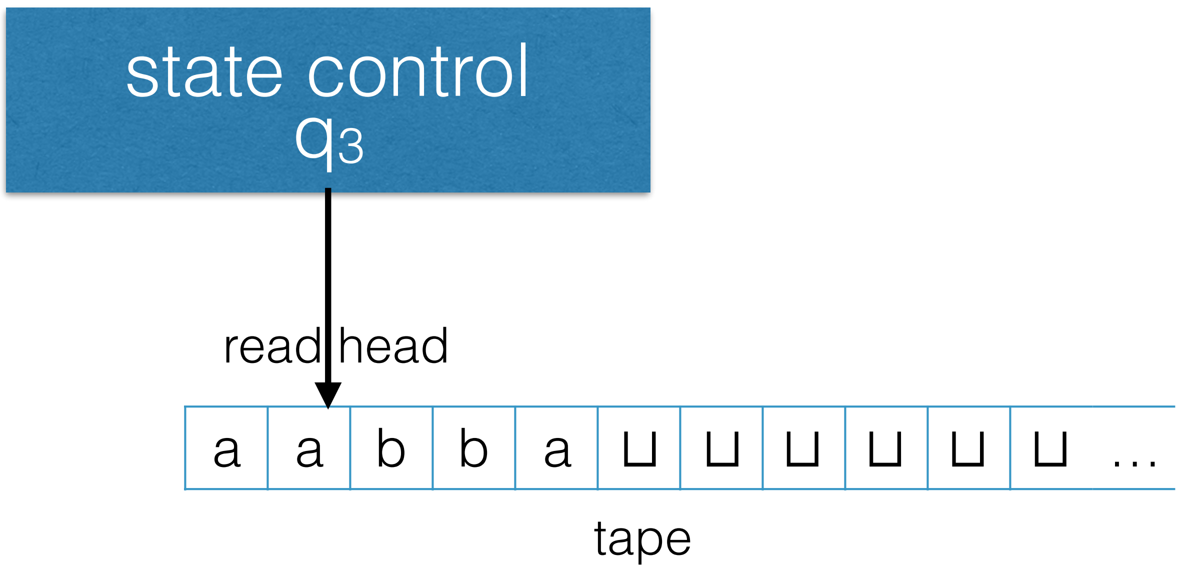
$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$



see complete formal definition 3.3 in text

Configuration of a TM

- current state
- current tape contents
- current head location



a q_3 a b b a

Possible outcomes of a TM on an input string

- halt
  - accept
  - reject
- loop

When does a TM accept a string?

if it eventually enters the q_{accept} state

recall that q_{accept} is a *halting* state

When does a TM reject a string?

1. if it eventually enters the q_{reject} state
2. if it never halts