Def: \( f(n) = O(t(n)) \)

\[ \exists c, n_0 \quad \forall n \geq n_0 \quad f(n) \leq c \cdot t(n) \]
Def: \( \text{TIME}(t(n)) \)
set of all languages decidable by
an \( \mathcal{O}(t(n)) \) time
single tape
deterministic TM

worst case analysis
\[ A = \{0^k1^k \mid k \in \mathcal{N}\} \]

\[ A \in \text{TIME}(n^2) \]
A = \{0^k1^k \mid k \in \mathbb{N}\}

M_1 =

“On input w:
1. Scan across tape and reject if of wrong form
2. Repeat if both 0s and 1s remain
   2.1. Scan across tape crossing off a single 0 and 1 each
3. If only 0s still remain (or 1s still remain) then reject else accept
A = \{0^k1^k \mid k \in \mathcal{N}\}

M_1 =

“On input w:

1. Scan across tape and reject if of wrong form

2. Repeat if both 0s and 1s remain
   
2.1. Scan across tape crossing off a single 0 and 1 each

3. If only 0s still remain (or 1s still remain) then reject else accept

O(n^2)
A = \{0^k1^k \mid k \in \mathcal{N}\}

M' =

“On input w:

1. Scan across tape and reject if of wrong form

2. Repeat if both 0s and 1s remain
   2.1. Scan across tape crossing off two 0s and 1s each

3. If only 0s still remain (or 1s still remain) then reject else accept
$A = \{0^k1^k \mid k \in \mathbb{N}\}$

$M' =$

“On input $w$:

1. Scan across tape and **reject** if of wrong form
2. Repeat if both 0s and 1s remain
   2.1. Scan across tape crossing off **two** 0s and 1s each
3. If only 0s still remain (or 1s still remain) then **reject** else **accept**

$O(n^2)$
\[ A = \{0^k1^k \mid k \in \mathbb{N}\} \]
A = \{0^k1^k \mid k \in \mathcal{N}\}

M_2 =

“On input w:
1. Scan across tape and reject if of wrong form
2. Repeat if both 0s and 1s remain
   2.1. Scan across tape to check if total remaining symbols is even or odd. If odd, reject
   2.2. Scan across tape and cross off every other 0 (starting with first) and every other 1 (starting with first)
3. If either 0s or 1s still remain then reject else accept
A = \{0^k1^k \mid k \in \mathbb{N}\}

M_2 =

“On input \textbf{w}:

1. Scan across tape and \textbf{reject} if of wrong form
2. Repeat if \textbf{both} 0s and 1s remain
   2.1. Scan across tape to check if total remaining symbols is even or odd. If odd, \textbf{reject}
   2.2. Scan across tape and cross off every other 0 (starting with first) \textbf{and} every other 1 (starting with first)
3. If either 0s or 1s still remain \textbf{then reject} else \textbf{accept}
A = \{0^k1^k \mid k \in \mathcal{N}\}

A \in \text{TIME}(n \log n)
Every $t(n)$ multitape TM has an equivalent $O(\cdot)$ single tape TM $t(n) \geq n$
starting configuration

\[
\begin{array}{c}
M \\
\downarrow \\
\text{a a b } \underbrace{\text{u u u u u u u u u}}_{\ldots}
\end{array}
\]

\[
\begin{array}{c}
\downarrow \\
\underbrace{\text{u u u u u u u u u}}_{\ldots}
\end{array}
\]

\[
\begin{array}{c}
\downarrow \\
\underbrace{\text{u u u u u u u u u}}_{\ldots}
\end{array}
\]

\[
\begin{array}{c}
S \\
\downarrow \\
\# \text{ a a b } \# \underbrace{\text{u u u u u u u u u}}_{\ldots}
\end{array}
\]

\[O(n)\]
intermediate configuration

$$M$$

$$O(t(n))$$

# a a b # b a # a b a a # # # ...
transition function

1. **remember** current (marked) symbols
2. **replace** marked symbols as indicated by M’s transition function
3. **move** marked symbols
   a. possibly **shift** tape contents right
Every \( t(n) \) multitape TM has an equivalent \( O(t(n)^2) \) single tape TM.
Every $t(n)$ nondeterministic TM has an equivalent $O(\cdot)$ single tape deterministic TM. 

$t(n) \geq n$
Nondeterministic TM: Decider
1. Depth of each node is $O(t(n))$
   \[
   \leq c \cdot t(n) = 2^{\log(c \cdot t(n))} = 2^{O(\log t(n))}
   \]

2. Each node has at most $b$ children

3. Total number of nodes is at most
   \[
   1 + b + \ldots + b^{t(n)} = O(b^{t(n)})
   \]
   \[
   = 2^{O(t(n))}
   \]

4. So running time is $O(t(n) b^{t(n)})$
   \[
   = 2^{O(t(n))} + O(\log t(n))
   \]
   \[
   = 2^{O(t(n))}
   \]
Every $t(n)$ nondeterministic TM has an equivalent $2^{O(t(n))}$ single tape deterministic TM $t(n) \geq n$
The Class $\mathbf{P}$

$$P = \bigcup_k \text{TIME}(n^k)$$
RELPRIME $\in P$