Def: \( f(n) = O(t(n)) \)

\[ \exists c, n_0 \quad \forall n \geq n_0 \quad f(n) \leq c \cdot t(n) \]
Def: \(\text{TIME}(t(n))\)
set of all languages decidable by an \(O(t(n))\) time single tape deterministic TM

worst case analysis
Every $t(n)$ multitape TM has an equivalent $O(t(n)^2)$ single tape TM

$t(n) \geq n$
Every $t(n)$ nondeterministic TM has an equivalent $2^{O(t(n))}$ single tape deterministic TM

$t(n) \geq n$
The Class $\mathbf{P}$

$$\mathbf{P} = \bigcup_{k} \text{TIME}(n^k)$$
“x divides y”

\[ x \mid y \text{ iff } y = kx \text{ for some integer } k \]
if $a = bq + r$
then $\gcd(a, b) = \gcd(b, r)$
RELPRIME = \{ \langle x, y \rangle \mid x \text{ and } y \text{ are relatively prime} \}

\langle 5, 7 \rangle \in \text{RELPRIME}?
\langle 21, 8 \rangle \in \text{RELPRIME}?
RELPRIME = \{ \langle x, y \rangle \mid x \text{ and } y \text{ are relatively prime} \}

RELPRIME \in P
G is a directed graph
Does it have a path from $s$ to $t$?
G is a directed graph
Does it have a path from s to t?

worst case analysis: $O(n^n)$
G is a directed graph
Does it have a path from $s$ to $t$?

Step 1: Mark $s$
$G$ is a directed graph
Does it have a path from $s$ to $t$?

Step 2: Repeat until no additional node is marked
1. Scan through all edges
2. Locate edge of form (marked, unmarked)
3. Mark unmarked node
G is a directed graph
Does it have a path from $s$ to $t$?
G is a directed graph
Does it have a path from s to t?

No additional node marked, so halt
Step 4: Check if t is marked.
No so reject.
$G$ is a directed graph
Does it have a path from $s$ to $t$?

Step 1: Mark $s$ $O(n)$
G is a directed graph
Does it have a path from s to t?

No additional node marked, so halt
Step 4: Check if t is marked.
No so reject.
G is a directed graph
Does it have a path from \( s \) to \( t \)?

**Step 2**: Repeat until no additional node is marked
1. Scan through all edges
2. Locate edge of form (marked, unmarked)
3. Mark unmarked node

\( \mathcal{O}(n) \)
\( \mathcal{O}(n^2) \)
PATH = \{ \langle G, s, t \rangle \mid G \text{ is a directed graph that has a path from } s \text{ to } t \}