The Class $\textbf{NP}$

class of languages that have polynomial time verifiers
A language is in $\text{NP}$ iff it is decided by some nondeterministic polynomial time $\text{TM}$.
A language is in \textbf{NP} iff it is decided by some \textit{nondeterministic polynomial time} \textbf{TM} has a polytime verifier.
If a language is decided by some **nondeterministic polynomial time** TM, then it is in **NP** has a polytime verifier.
In other words, construct verifier $V$ such that

- $V$ is a **decider**
- $V$ takes as input $\langle w, c \rangle$

- **For every** $w \in A$
- **there is some** $c \in \Sigma^*$
  - $V$ accepts $\langle w, c \rangle$

- **For every** $w' \not\in A$
- **there is no** $c \in \Sigma^*$
  - $V$ accepts $\langle w', c \rangle$
In other words, construct verifier $V$ such that

- $V$ is a **decider**
- $V$ takes as input $\langle w, c \rangle$

- For **every** $w \in A$
- there is **some** $c \in \Sigma^*$
  - $V$ accepts $\langle w, c \rangle$

- For **every** $w' \notin A$
- for **every** $c \in \Sigma^*$
  - $V$ rejects $\langle w', c \rangle$
Proof Idea: interpret c as address label
Given: nondeterministic polytime TM $N$ that decides $A$
Construct: polytime verifier $V$ for $A$

$V$: On input $\langle w, c \rangle$

1. *Deterministically* simulate the branch of $N$ indicated by $c$
   1.1. If it accepts, *accept*
   1.2. If it rejects, *reject*
A language is in \textbf{NP} iff it is decided by some nondeterministic polynomial time TM has a polytime verifier.

\[
\text{NP} = \bigcup_k \text{NTIME}(n^k)
\]
HAMPATH = \{ \langle G, s, t \rangle \mid G \text{ has a Hamiltonian path from } s \text{ to } t \}\}

HAMPATH ∈ NP
coNP
complement of languages in NP

HAMPATH
CLIQUE = \{ \langle G, k \rangle \mid G \text{ is an undirected graph with a } k\text{-clique} \}\}
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CLIQUE \in \text{NP}
CLIQUE = \{ \langle G,k \rangle \mid G \text{ is an undirected graph with a } k\text{-clique} \}\}

\textbf{certificate?}

\textbf{nondeterministic choice?}
SUBSET-SUM = 
\{ \langle S, t \rangle \mid S = \{x_1, x_2, \ldots, x_n\}, \{y_1, y_2, \ldots, y_k\} \subseteq \{x_1, x_2, \ldots, x_n\} \\
\text{and } \sum y_i = t \}

S=\{4, -11, 16, 21, 27\}
\langle S, 5 \rangle \in \text{SUBSET-SUM?}
\langle S, 15 \rangle \in \text{SUBSET-SUM?}
SUBSET-SUM = 
\{ \langle S, t \rangle \mid S = \{x_1, x_2, \ldots, x_n\}, \\
\{y_1, y_2, \ldots, y_k\} \subseteq \{x_1, x_2, \ldots, x_n\} \\
\text{and } \Sigma y_i = t \}

\text{SUBSET-SUM } \in \text{ NP}
SUBSET-SUM = 
\{ \langle S, t \rangle \mid S = \{x_1, x_2, \ldots, x_n\}, \\
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and \sum y_i = t \}\}

certificate?

nondeterministic choice?
SAT = \{ \langle \Phi \rangle \mid \Phi \text{ is a satisfiable Boolean formula} \}

SAT \in NP

certificate?

nondeterministic choice?
The Class \textbf{EXPTIME}

\[
\text{EXPTIME} = \bigcup_{k} \text{TIME}(2^{n^k})
\]
All languages
Def: polynomial time computable function

function \( f: \Sigma^* \rightarrow \Sigma^* \)
is polynomial time computable iff
on input \( w \) some TM \( M \) halts with exactly \( f(w) \) on its tape in \( O(n^k) \) time
Def: polynomial time mapping reducible

\[ A \leq_p B \]

iff

\[ \exists \text{ polynomial time computable function } f: \Sigma^* \to \Sigma^* \]

\[ \forall w \in \Sigma^* \quad w \in A \iff f(w) \in B \]
If

\[ A \leq_P B \text{ and } B \in P \]

then

\[ A \in P \]