We design a class of proper scoring rules for infinite outcome spaces based on maximum entropy distributions that elicit statistics of the data. We define a cost function based prediction market based on this scoring rule and consider the semantics of information aggregation in this market. We also explicitly characterize the interaction between markets on related events by drawing on results from graphical models. In one instance, we are able to show that a trader in such a market behaves as if he were implementing a learning algorithm, even though his incentives are purely financial.

**Generalized LMSR**

Idea: Elicit summary information $f(x)$ e.g., $f(x) = (x, x^2)$ agent believes $m = E[f(x)]$ for any belief distribution $r$

Score is $\log p(x; m)$ for the maxent distribution $p$ $\max_{m} \cdot \int p(x) \log p(x) \; dx \quad \text{s.t.} \; E[f(x)] = m$

**How to elicit beliefs on large (infinite) outcome spaces**

**Expontial Family Prediction Market**

- **Exponential Families**
  - sufficient statistics
  - log partition function
  - natural parameters
  - mean parameter

**Cost Function Prediction Market**

- contract payoff
- cost function
- outstanding share vector
- market prices

**Gaussian Market**

- outcome drawn from a normal distribution with unknown mean and variance

**Risk Averse Traders**

Potential games-like argument $U_\alpha(w) = -\frac{1}{\beta} \exp(-\alpha w)$

$U_1(\delta) - U_1(\delta_{i-1}, \delta_i') = F(\delta) - F(\delta_{i-1}, \delta_i')$

Nash Equilibrium $\iff$ local optimum

$F(\delta) = \psi(\theta + \sum_i \delta_i') + \sum_i \frac{1}{\alpha_i} \psi(\theta_i - a_i \delta_i)$

Equilibrium state is a convex combination of current market state and weighted agent beliefs

**Bayesian Traders**

- $\nabla C(\theta) = \nu$
- $\nabla C(\theta + \delta) = \frac{m \nu + m \mu}{m + \sum_i m_i \mu_i}$

- Risk neutral
- $\mu = \sum_i m_i \bar{x}_i$
- $\nabla C(\theta + \delta) = \frac{m \nu + \sum_i m_i \mu_i}{m + \sum_i m_i \mu_i}$

Final market prices average of initial market prices and mean parameter beliefs of all agents

**Partially informed traders**

- Current market estimate $m$
- Optimal trade for a trader with partial data

Optimal trade if interested only in $x_1$

**EM algorithm**

- $E$ step $\mu^{t+1} \leftarrow E_{p_0}[f(x_1, x_2, x_3, x_4, x_5)]$
- $M$ step $\theta^{t+1} \leftarrow \arg \max \left\{ \mu^{t+1} \cdot \theta - C(\theta) \right\}$

**Joint Markets**

- Joint distribution GMRF $p_\theta$
- Payoff of contracts are $x_0, x_1, x_2, x_3, x_4, x_5$

Effect on $p(x_1)$ for purchase of $\delta$ shares in $\theta_i$

$\rho_i = \rho_{i+1}, \delta_i = \frac{\rho_i}{c_{i+1}, \theta_i}$