Wrapping up Recursion

Announcements

- Lab 10 (recursion) posted
  - Due Saturday at midnight

Today's plan

- Catalogue several different forms of recursion
- Recursion gotchas
- Recursive binary search

Recursing over ints

- Typical base case: n == 0 or n == 1
- Typical general case: use fn(n-1) in solution for fn(n)
- Practice sheet: #1, #4

Recursing over ints

```python
def factorial(n):
  if n == 1:
    return 1
  else:
    return n * factorial(n-1)
```

Recursing over lists

- Typical base case: len(L) == 0 or len(L) == 1
- Typical general case: Use L[0] and fn(L[1:]) to solve fn(L)
- Practice sheet: #2, #3, #9, #10

Find the bug

```
def recursiveLinearSearch(x, L):
  if len(L) == 0:
    return False
  else:
    if L[0] == x:
      return True
    else:
      recursiveLinearSearch(x, L[1:])
```

Recursing over lists

```
def recursiveLinearSearch(x, L):
  if len(L) == 0:
    return False
  else:
    if L[0] == x:
      return True
    else:
      return recursiveLinearSearch(x, L[1:])
```

Recursing over strings

- Typical base case: s == “”
- Typical general case: use s[0] and fn(s[1:]) to solve fn(s)
- Practice sheet: #6, #7, #8

Recursing over strings

```
def countLetter(s, l):
  if s == "":
    return 0
  elif s[0] == l:
    return 1 + countLetter(s[1:], l)
  else:
    return countLetter(s[1:], l)
```

How to approach recursion

1. Identify what we're recursing over. For this example, let's imagine it's a string and our function is called foo(s).

2. Solve the base case, foo(“”).

3. Imagine you have a working version of foo. Ask yourself what foo(s[1:]) would return. Combine it with s[0] to figure out the return value for foo(s).

4. Don't forget the return statements

Multiple general cases

- Often within the general case, we want to examine n, L[0], s[0], etc. in an if statement.

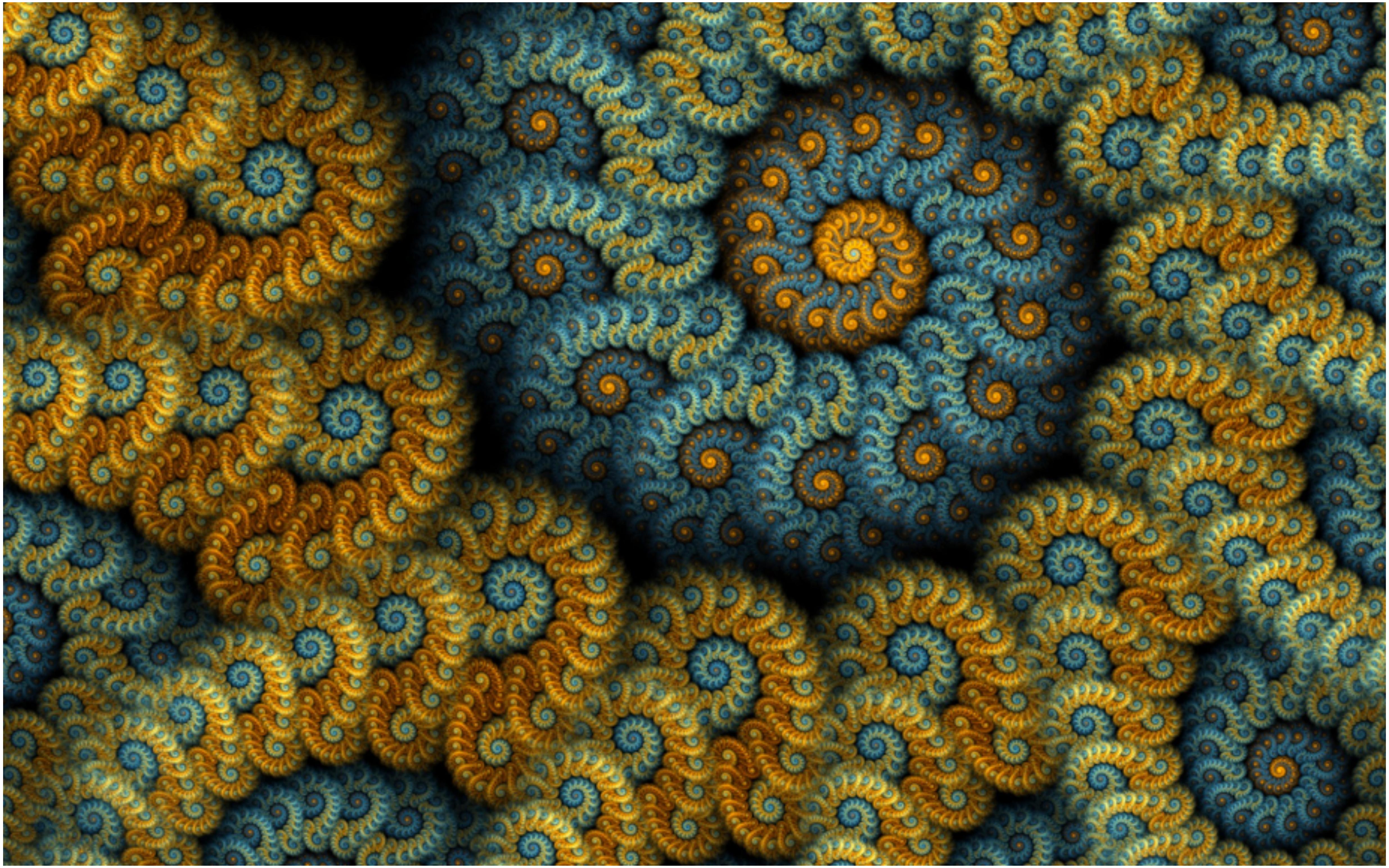
```python
def countHeads(n):
  if n == 0:
    return 0
  else:
    flip = choice(['heads', 'tails'])
    if flip == 'heads':
      return 1 + countHeads(n-1)
    else:
      return countHeads(n)
```

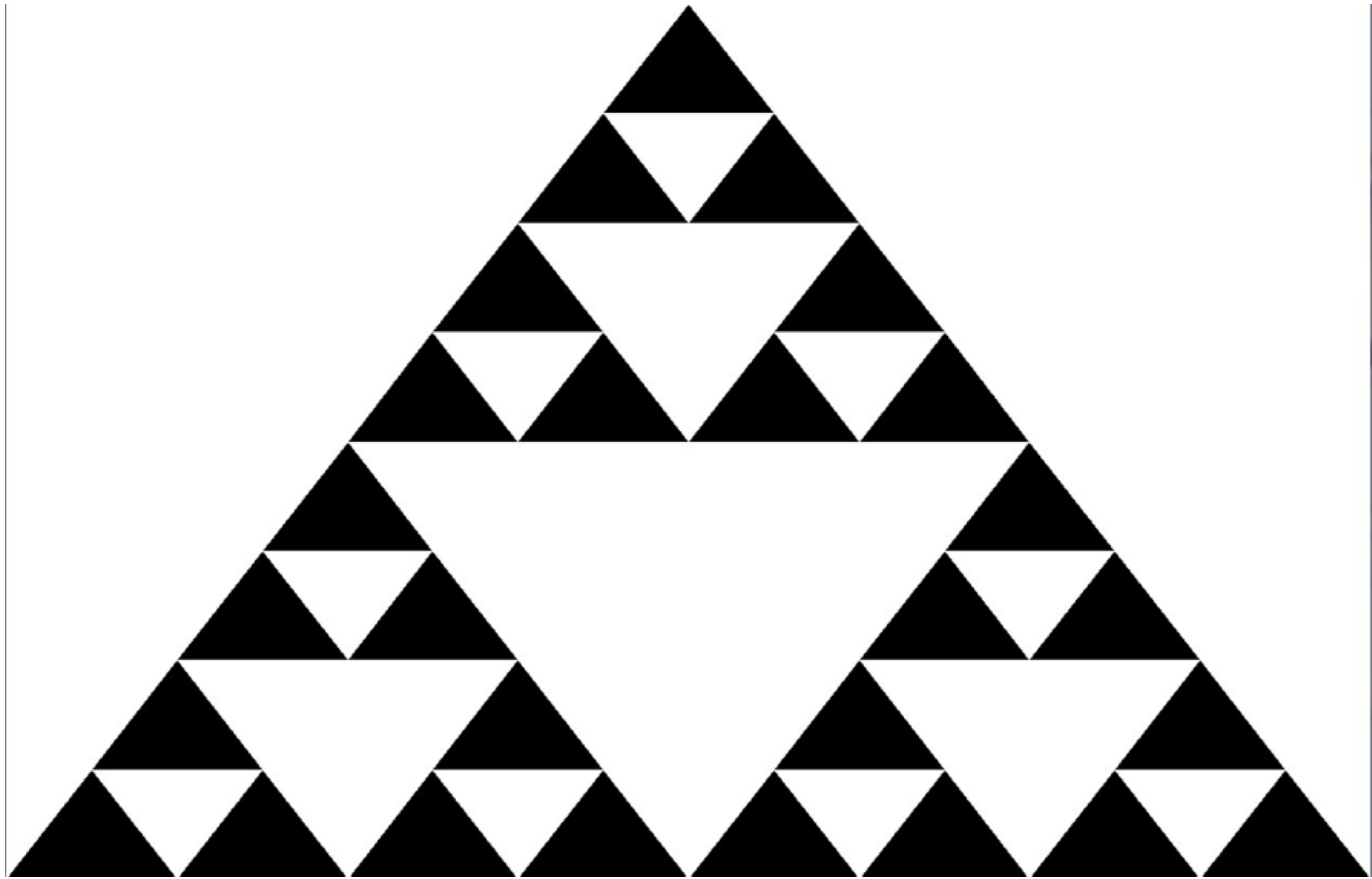
Recursive graphics

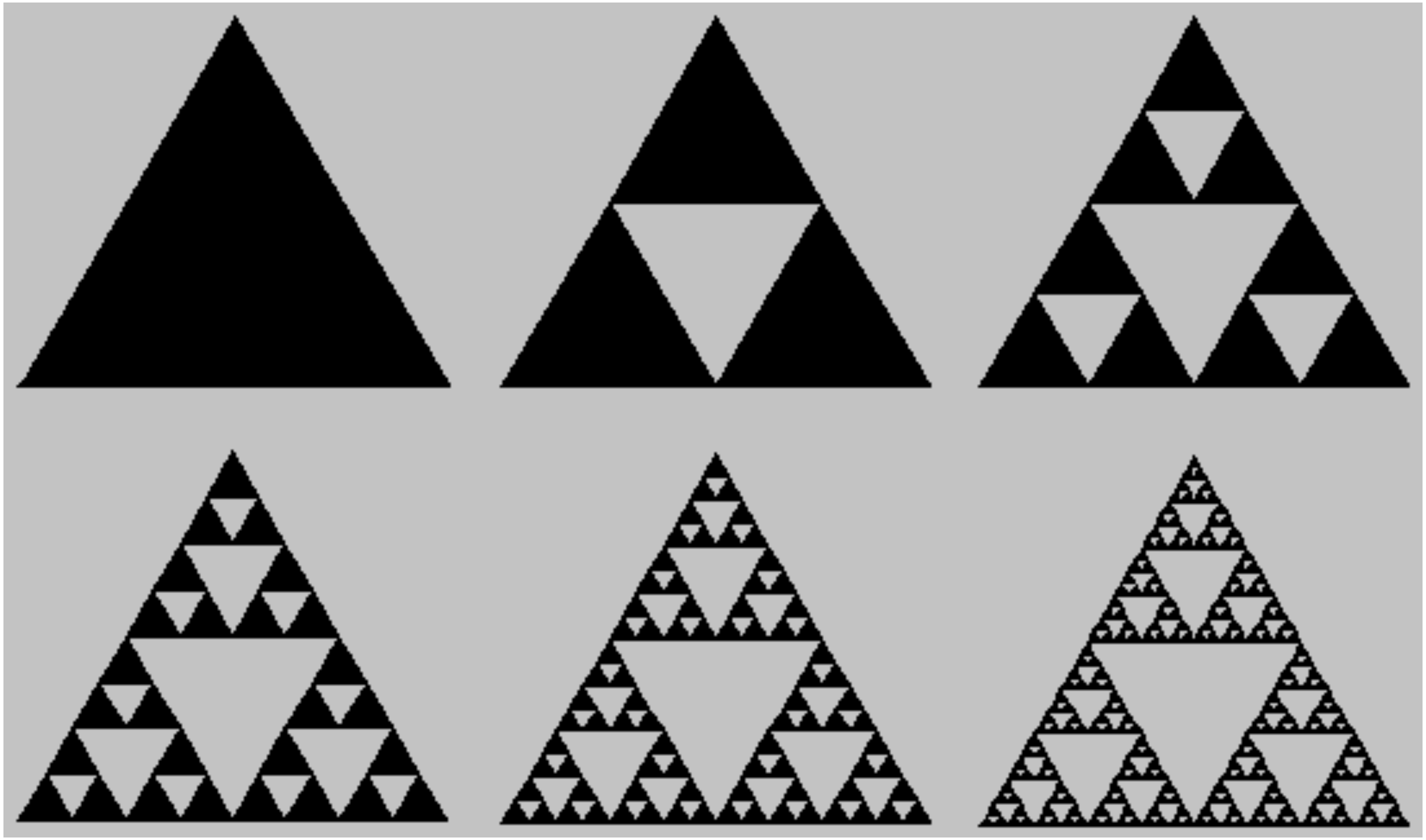
- **Fractals** are self-repeating images. You can zoom in on a fractal and see a sub-image that closely resembles the original image.

- They appear in nature: trees, lightning, river tributaries…

- When we generate a fractal using computer graphics, it is natural to use recursion.







Multiple recursive calls

- Solve the problem with solutions to multiple smaller sub-problems:
  - Merge sort
  - Fractals
- Exponential growth

Returning new lists

```
def reverse(L):
  if len(L) == 1:
    return L
  else:
    return reverse(L[1:]) + L[:1]
```

Modifying a list in place

- Do the recursion over an integer that represents the index.
- The list and the index are both parameters.
- Use a wrapper function to avoid passing in the initial index.

Modifying lists in-place

```
def squareOddIndicesH(L, index):
  if index == len(L):
    return
  else:
    if index % 2 == 1:
      L[index] = L[index]**2
    squareOddIndicesH(L, index+1)

def squareOddIndices(L):
  squareOddIndicesH(L, 0)
```

Recursion gotchas

- If you forget the base case, the function will continue calling itself indefinitely, until the stack reaches its maximum size. This also happens if your sub-problem is the same size as your original problem, e.g. foo(n) instead of foo(n-1).
  - `RuntimeError: maximum recursion depth exceeded`
- With functions that are called for their return value, it is easy to forget the ‘return’

Recursive binary search

- Pass ‘lo’ and ‘hi’ as additional parameters.
- Update the range of indices when you make the recursive call.
- Recursion makes sense here because binary search is repeatedly breaking the search down into a binary search on a smaller list.

```python
def binarySearchH(x, L, lo, hi):
  if lo > hi:
    return False
  else:
    mid = (lo+hi)/2
    if L[mid] == x:
      return True
    elif L[mid] < x:
      return binarySearchH(x, L, mid+1, hi)
    else:
      return binarySearchH(x, L, lo, mid-1)

def binarySearch(x, L):
  return binarySearchH(x, L, 0, len(L)-1)
```

See you Wednesday!