CS 31: Intro to Systems
Binary Representation

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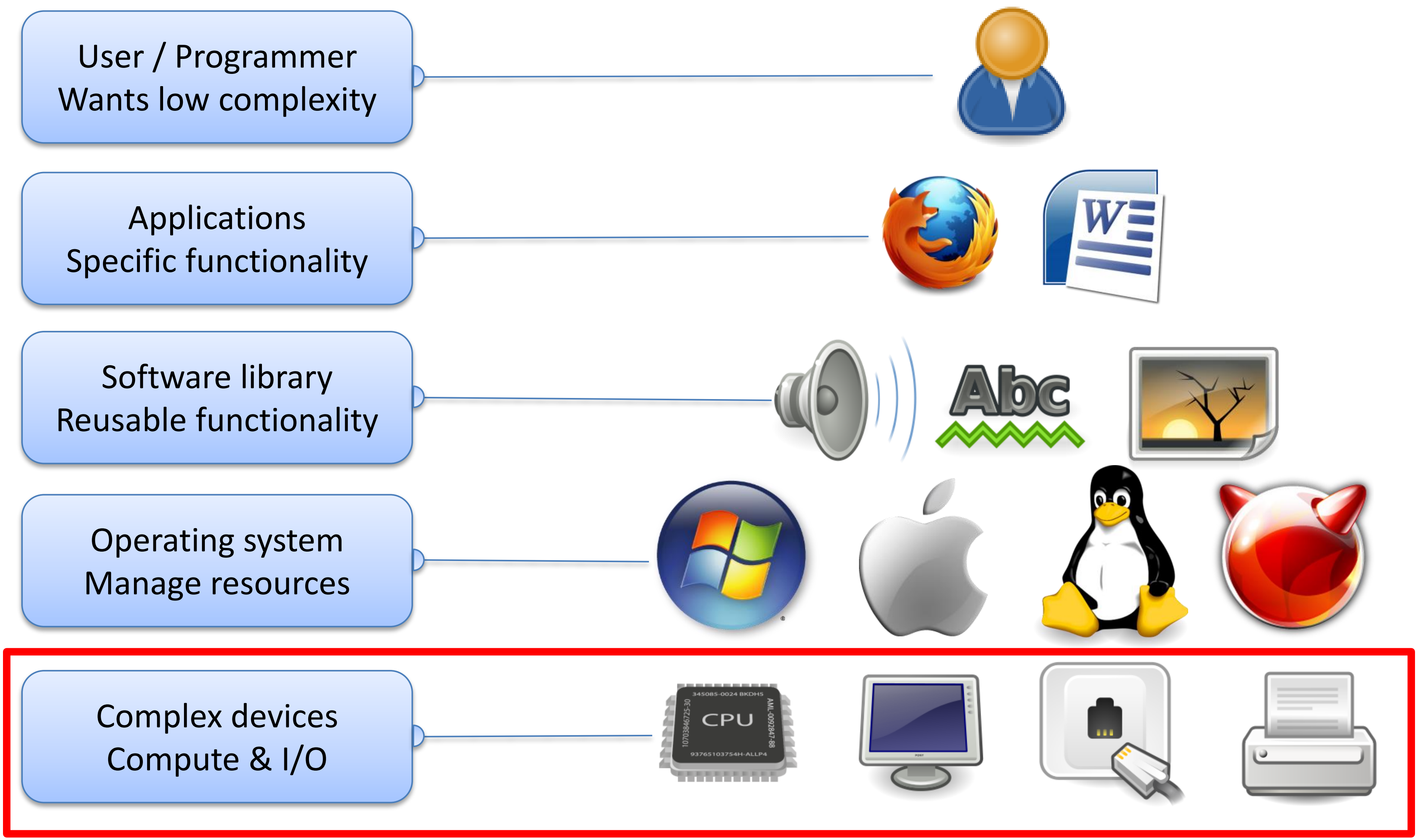
Reading Quiz

Abstraction



Today

- Number systems and conversion
- Data types and storage:
  - Sizes
  - Representation
  - Signedness

You can view binary file contents

xxd (or hexdump –C) to view binary file values:

xxd a.out   # a binary executable file

```
Address:  value of the next 16 bytes in memory
0000000: 7f45 4c46 0201 0100 0000 0000 0000 0000
0000010: 0200 3e00 0100 0000 3007 4000 0000 0000
0000020: 4000 0000 0000 0000 084d 0000 0000 0000
  …
```

(these weird numbers (f,c,e, …), are hexidecimal digits)

xxd myprog.c   # binary ascii encoding of C source:

```
0000000: 2369 6e63 6c75 6465 3c73 7464 696f 2e68
         #i   nc   lu   de   <s   td   io   .h
0000010: 3e0a 696e 7420 6d61 696e 2829 207b 0a20
         >\n  in   t    ma   in   ()    {  \n
…
```

Data Storage

- Lots of technologies out there:
  - Magnetic (hard drive, floppy disk)
  - Optical (CD / DVD / Blu-Ray)
  - Electronic (RAM, registers, …)

- Focus on electronic for now
  - We'll see (and build) digital circuits soon

- Relatively easy to differentiate two states
  - Voltage present
  - Voltage absent

Bits and Bytes

- Bit: a 0 or 1 value (binary)
  - HW represents as two different voltages
    - 1: the presence of voltage (high voltage)
    - 0: the absence of voltage (low voltage)

- Byte: 8 bits, the smallest addressable unit

Memory:	01010101	10101010	00001111	…
(address)	[0]	[1]	[2]	…

- Other names:
  - 4 bits: Nibble
  - "Word": Depends on system, often 4 bytes

How many unique values can we represent with 9 bits?

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

A. 18
B. 81
C. 256
D. 512
E. Some other number of values.

How many values?

1 bit:	0				1			
2 bits:	0 0		0 1		1 0		1 1	
3 bits:	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1

4 bits:

0 0 0 0	0 0 0 1	0 0 1 0	0 0 1 1	16 values
0 1 0 0	0 1 0 1	0 1 1 0	0 1 1 1	
1 0 0 0	1 0 0 1	1 0 1 0	1 0 1 1	
1 1 0 0	1 1 0 1	1 1 1 0	1 1 1 1	

N bits: 2^N values

C types and their (typical!) sizes

- 1 byte: char, unsigned char
- 2 bytes: short, unsigned short
- 4 bytes: int, unsigned int, float
- 8 bytes: long long, unsigned long long, double
- 4 or 8 bytes: long, unsigned long

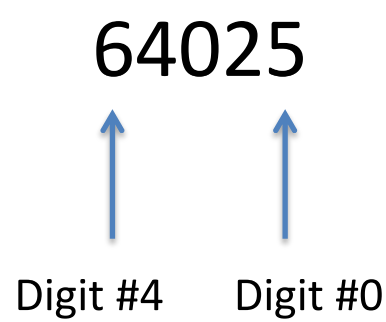
```
unsigned long v1;
short s1;
long long ll;

printf("%lu %lu %lu\n", sizeof(v1), sizeof(s1),
          sizeof(ll));  // prints out number of bytes
```

How do we use this storage space (bits) to represent a value?

Let’s start with what we know…

- Decimal number system (Base 10)
- Sequence of digits in range [0, 9]



What is the significance of the N^{th} digit number in this number system? What does it contribute to the overall value?

64025

Digit #4 d_4

Digit #0 d_0

A. $d_N * 1$
B. $d_N * 10$
C. $d_N * 10^N$
D. $d_N * N^{10}$
E. $d_N * 10^{d_N}$

Consider the meaning of d_3 (the value 4) above.
What is it contributing to the total value?

Positional Notation

- The meaning of a digit depends on its position in a number.
- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ in base b represents the value

$$d_n * b^n + d_{n-1} * b^{n-1} + \ldots + d_2 * b^2 + d_1 * b^1 + d_0 * b^0$$

Decimal: Base 10

- Used by humans
- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1,2,3,4,5,6,7,8,9}, represents the value

$$d_n * 10^n + d_{n-1} * 10^{n-1} + \ldots + d_2 * 10^2 + d_1 * 10^1 + d_0 * 10^0$$

64025 =

$6 * 10^4$	+	$4 * 10^3$	+	$0 * 10^2$	+	$2 * 10^1$	+	$5 * 10^0$
60000	+	4000	+	0	+	20	+	5

Binary: Base 2

- Used by computers

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1}, represents the value

$$d_n * 2^n + d_{n-1} * 2^{n-1} + \ldots + d_2 * 2^2 + d_1 * 2^1 + d_0 * 2^0$$

What is the value of 110101 in decimal?

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1}, represents the value

  $$d_n * 2^n + d_{n-1} * 2^{n-1} + \ldots + d_2 * 2^2 + d_1 * 2^1 + d_0 * 2^0$$

A. 26

B. 53

C. 61

D. 106

E. 128

Other (common) number systems.

- Base 10: decimal
- Base 2: binary

- Base 16: hexadecimal (memory addresses)
- Base 8: octal
- Base 64 (Commonly used on the Internet, e.g. email attachments).

Hexadecimal: Base 16

- Indicated by prefacing number with 0x
- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}, represents the value

$$d_n * 16^n + d_{n-1} * 16^{n-1} + \ldots + d_2 * 16^2 + d_1 * 16^1 + d_0 * 16^0$$

What is the value of 0x1B7 in decimal?

A. 397
B. 409
C. 419
D. 437
E. 439

$16^2 = 256$

Hexadecimal: Base 16

- Indicated by prefacing number with 0x
- Like binary, base is power of 2
  - Fewer digits to represent same value
- Each digit is a "nibble", or half a byte

- A number, written as the sequence of digits $d_n d_{n-1} \ldots d_2 d_1 d_0$ where d is in {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}, represents the value

$$d_n * 16^n + d_{n-1} * 16^{n-1} + \ldots + d_2 * 16^2 + d_1 * 16^1 + d_0 * 16^0$$

Each hex digit is a “nibble”

Hex digit:16 values, $2^4 = 16$ -> 4 bits / digit

0x 1 B 7

Four-bit value: 1

Four-bit value: B (decimal 11)

Four-bit value: 7

In binary:	0001	1011	0111
	1	B	7

Converting Decimal -> Binary

- Two methods:
  - division by two remainder
  - powers of two and subtraction

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
     set b_i to 1
  if D is even
     set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:    D = b          example: D = 105          a0 = 1
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
      set b_i to 1
  if D is even
      set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:    D = b          example: D = 105        a0 = 1
         D/2 = b/2               D =  52        a1 = 0
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
  if D is odd
      set b_i to 1
  if D is even
      set b_i to 0
  i++
  D = D/2
```

Example: Converting 105

```
idea:     D = b           example: D = 105        a0 = 1
        D/2 = b/2                  D =  52        a1 = 0
        D/2 = b/2                  D =  26        a2 = 0
        D/2 = b/2                  D =  13        a3 = 1
        D/2 = b/2                  D =   6        a4 = 0
        D/2 = b/2                  D =   3        a5 = 1
          0 =   0                  D =   1        a6 = 1
                                   D =   0        a7 = 0

                                    105    =   01101001
```

Method 2

- $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$
- To convert 105:
  - Find largest power of two that's less than 105 (64)
  - Subtract 64 (105 – 64 = 41), put a 1 in d_6
  - Subtract 32 (41 – 32 = 9), put a 1 in d_5
  - Skip 16, it's larger than 9, put a 0 in d_4
  - Subtract 8 (9 – 8 = 1), put a 1 in d_3
  - Skip 4 and 2, put a 0 in d_2 and d_1
  - Subtract 1 (1 – 1 = 0), put a 1 in d_0 (Done)

__ __ __ __ __ __ __

What is the value of 357 in binary?

A. 101100011
B. 101100101
C. 101101001
D. 101110101
E. 110100101

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$,
$2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$

So far: Unsigned Integers

- With N bits, can represent values: 0 to 2^n-1
- We can always add 0's to the front of a number without changing it:

10110= 010110 = 00010110 = 0000010110

- 1 byte: char, unsigned char
- 2 bytes: short, unsigned short
- 4 bytes: int, unsigned int, float
- 8 bytes: long long, unsigned long long, double
- 4 or 8 bytes: long, unsigned long

Representing Signed Values

- One option (used for floats, <u>NOT integers</u>)
  - Let the first bit represent the sign
  - 0 means positive
  - 1 means negative

- For example:
  - 0101 -> 5
  - 1101 -> -5

- Problem with this scheme?

Floating Point Representation

1 bit for sign   sign | exponent | fraction |
8 bits for exponent
23 bits for precision

$$value = (-1)^{sign} * 1.fraction * 2^{(exponent-127)}$$

let's just plug in some values and try it out

```
0x40ac49ba: 0 10000001   01011000100100110111010
      sign = 0 exp = 129   fraction = 2902458
```

$$= 1*1.2902458*2^2 = 5.16098$$

I don’t expect you to memorize this

Up Next: Binary Arithmetic