CS 31: Intro to Systems
Binary Representation

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Reading Quiz

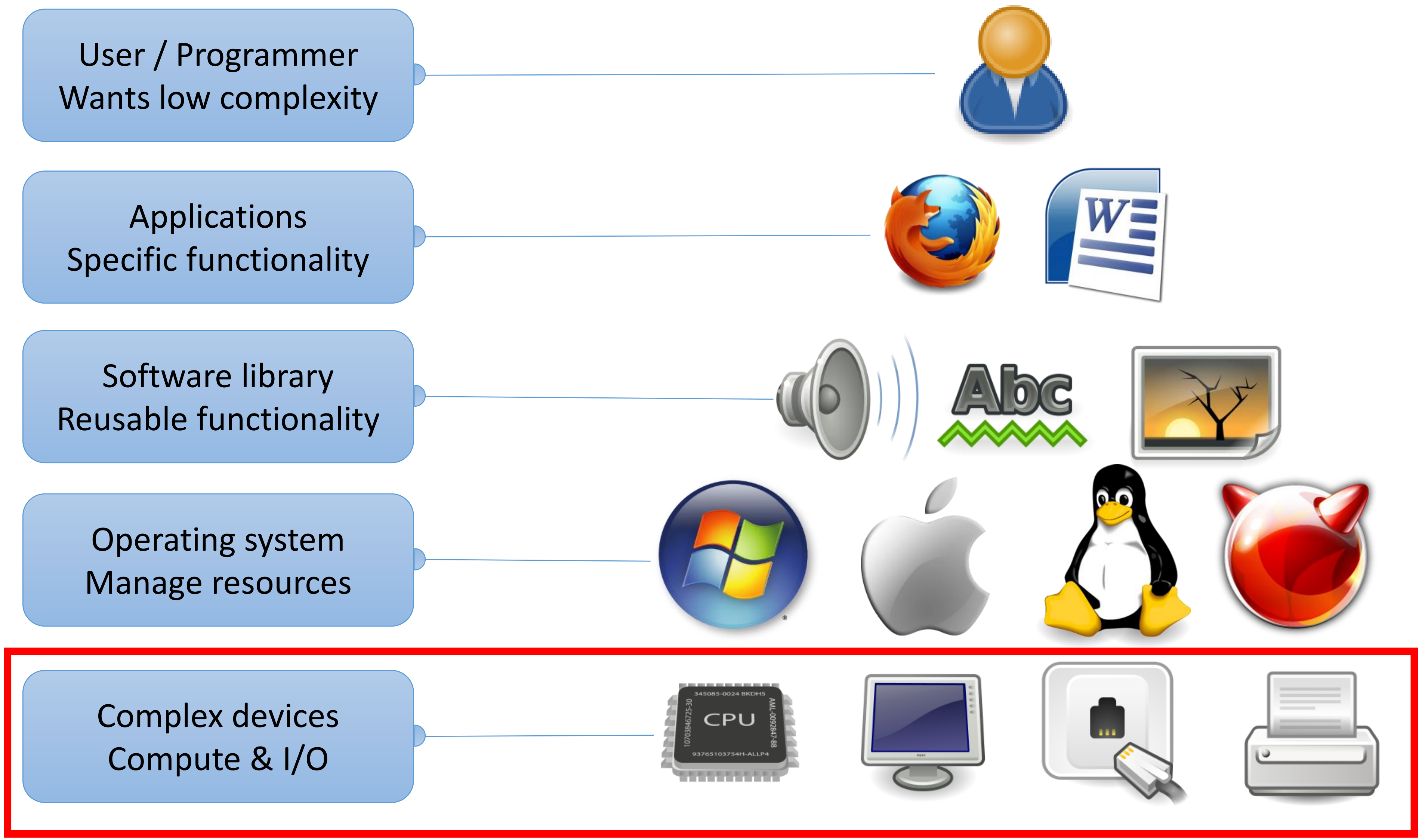
- Note the red border!
- 1 minute per question
- No talking, no laptops, phones during the quiz

Today

- Number systems and conversion

- Data types and storage:
  - Sizes
  - Representation
  - Signedness

Abstraction



Data Storage

- Lots of technologies out there:
  - Magnetic (hard drive, floppy disk)
  - Optical (CD / DVD / Blu-Ray)
  - Electronic (RAM, registers, …)
- Focus on electronic for now
  - We'll see (and build) digital circuits soon
- Relatively easy to differentiate two states
  - Voltage present
  - Voltage absent

Bits and Bytes

- Bit: a 0 or 1 value (binary)
  - HW represents as two different voltages
    - 1: the presence of voltage (high voltage)
    - 0: the absence of voltage (low voltage)

- Byte: 8 bits, the smallest addressable unit

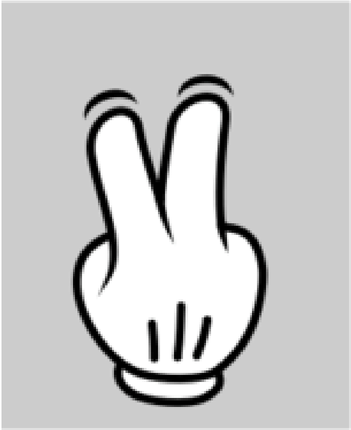
Memory:	01010101	10101010	00001111	…
(address)	[0]	[1]	[2]	…

- Other names:
  - 4 bits: Nibble
  - "Word": Depends on system, often 4 bytes

Files

Sequence of bytes… nothing more, nothing less

Binary Digits (BITs)



- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

Discussion question

- Green border

- Recall the sequence
  - Answer individually (room quiet)
  - Discuss in your group (room loud)
  - Answer as a group
  - Class-wide discussion

How many unique values can we represent with 9 bits? Why?

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

A. 18
B. 81
C. 256
D. 512
E. Some other number of values.

How many values?

1 bit: 0 1

How many values?

1 bit:

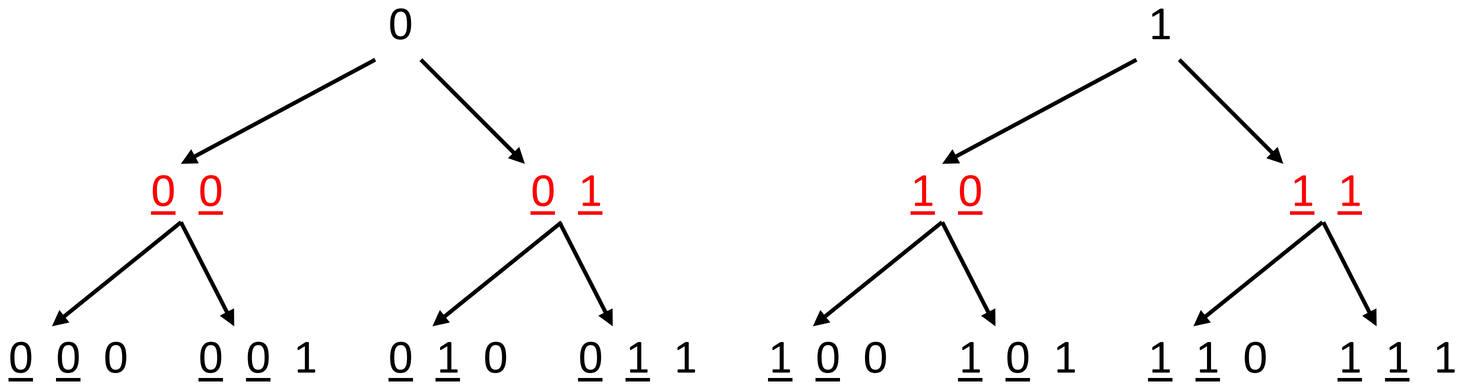
2 bits:

How many values?

1 bit:

2 bits:

3 bits:



How many values?

1 bit: 0 1

2 bits: 0 0 0 1 1 0 1 1

3 bits: 0 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1 1 1

4 bits:

0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 1 16 values
0 1 0 0 0 1 0 1 0 1 1 0 0 1 1 1

1 0 0 0 1 0 0 1 1 0 1 0 1 0 1 1
1 1 0 0 1 1 0 1 1 1 1 0 1 1 1 1

N bits: 2^N values

C types and their (typical!) sizes

- 1 byte: char, unsigned char
- 2 bytes: short, unsigned short
- 4 bytes: int, unsigned int, float
- 8 bytes: long long, unsigned long long, double
- 4 or 8 bytes: long,

```
unsigned long v1;
short s1;
long long ll;

// prints out number of bytes
printf(“%lu %lu %lu\n”, sizeof(v1), sizeof(s1), sizeof(ll));
```

WARNING: These sizes are **NOT** a guarantee. Don't always assume that every system will use these values!

How do we use this storage space (bits) to represent a value?

Let's start with what we know…

- Digits 0-9
- <u>Positional numbering</u>
- Digits are composed to make larger numbers
- Known as Base 10 representation

Decimal number system (Base 10)

- Sequence of digits in range [0, 9]

64025

Digit #0: 1’s place, “least significant digit”

Digit #1: 10’s place

Digit #4: “most significant digit”

What is the significance of the N^{th} digit number in this number system? What does it contribute to the overall value?

64025

Digit #4 (d_4) Digit #0 (d_0)

A. $d_N * 1$
B. $d_N * 10$
C. $d_N * 10^N$
D. $d_N * N^{10}$
E. $d_N * 10^{d_N}$

Consider the meaning of d_3 (the value 4) above.
What is it contributing to the total value?

Decimal: Base 10

A number, written as the sequence of N digits,

$$d_{n-1} \ldots d_2\, d_1\, d_0$$

where d is in {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, represents the value:

$$[d_{n-1} * 10^{n-1}] + [d_{n-2} * 10^{n-2}] + \ldots + [d_1 * 10^1] + [d_0 * 10^0]$$

64025 =

$6 * 10^4$ +	$4 * 10^3$ +	$0 * 10^2$ +	$2 * 10^1$ +	$5 * 10^0$
60000 +	4000 +	0 +	20 +	5

Generalizing: Base b

- The meaning of a digit depends on its position in a number.

A number, written as the sequence of N digits,

$$d_{n-1} \ldots d_2\, d_1\, d_0$$

in base b represents the value:

$$[d_{n-1} * b^{n-1}] + [d_{n-2} * b^{n-2}] + \ldots + [d_2 * b^2] + [d_1 * b^1] + [d_0 * b^0]$$

Base 10: $[d_{n-1} * 10^{n-1}] + [d_{n-2} * 10^{n-2}] + \ldots + [d_1 * 10^1] + [d_0 * 10^0]$

Binary: Base 2

- Used by computers to store digital values.

- Indicated by prefixing number with **0b**

- A number, written as the sequence of N digits, $d_{n-1}\ldots d_2 d_1 d_0$, where d is in {0,1}, represents the value:

$$[d_{n-1} * 2^{n-1}] + [d_{n-2} * 2^{n-2}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

What is the value of 0b110101 in decimal?

- A number, written as the sequence of N digits $d_{n-1}...d_2d_1d_0$ where d is in {0,1}, represents the value:

$$[d_{n-1} * 2^{n-1}] + [d_{n-2} * 2^{n-2}] + ... + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

A. 26
B. 53
C. 61
D. 106
E. 128

One more binary example…

Most significant bit → 10001111 ← Least significant bit

7 6 5 4 3 2 1 0

Representation: $1 \times 2^7 + 0 \times 2^6 \ldots + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$128 + \qquad + 8 + 4 + 2 + 1$

10001111 = 143

Other (common) number systems

- Base 10: decimal
- Base 2: binary

- Base 16: hexadecimal
- Base 8: octal
- Base 64

Hexadecimal: Base 16

- Indicated by prefixing number with **0x**

A number, written as the sequence of N digits,

$$d_{n-1}\dots d_2 d_1 d_0,$$

where d is in {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}, represents:

$$[d_{n-1} * 16^{n-1}] + [d_{n-2} * 16^{n-2}] + \dots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$$

What is the value of 0x1B7 in decimal?

$[d_{n-1} * 16^{n-1}] + [d_{n-2} * 16^{n-2}] + \ldots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$

(Note: $16^2 = 256$)

A. 397
B. 409
C. 419
D. 437
E. 439

DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Important Point…

- You can represent the same value in a variety of number systems or bases.

- It's **all** stored as binary in the computer.
  - Presence/absence of voltage.

Other (common) number systems.

- Base 2: How data is stored in hardware.
- Base 8: Used to represent file permissions.
- Base 10: Preferred by people.
- Base 16: Convenient for representing memory addresses.
- Base 64: Commonly used on the Internet, (e.g. email attachments).

It's **all** stored as binary in the computer.

Different representations (or visualizations) of the same information!

Hexadecimal: Base 16

- Fewer digits to represent same value
  - Same amount of information!
- Like binary, the base is power of 2
- Each digit is a “nibble”, or half a byte.

Each hex digit is a “nibble”

- One hex digit: 16 possible values (0-9, A-F)
- $16 = 2^4$, so each hex digit has exactly four bits worth of information.
- We can map each hex digit to a four-bit binary value.
  (helps for converting between bases)

Each hex digit is a “nibble”

Example value: 0x1B7

Four-bit value: 1

Four-bit value: B (decimal 11)

Four-bit value: 7

In binary:	0001	1011	0111
	1	B	7

Hexadecimal ↔ Binary Conversion

- Bit patterns as base-16 numbers
- Convert binary to hexadecimal: by splitting into groups of 4 bits each.

Example:

0b0011 1100 1010 1101 1011 0011 = 0x3CADB3

Bin	0011	1100	1010	1101	1011	0011
Hex	3	C	A	D	B	3

Converting Decimal -> Binary

- Two methods:
  - division by two remainder
  - powers of two and subtraction

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
   if D is odd
          set b_i to 1
   if D is even
          set b_i to 0
   i++
   D = D/2
```

Example: Converting 105

idea:

example: D = 105 b_0 = 1

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
   if D is odd
          set b_i to 1
   if D is even
          set b_i to 0
   i++
   D = D/2
```

Example: Converting 105

```
idea:     D            example: D = 105         b_0 = 1
          D = D/2               D =  52         b_1 = 0
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
    if D is odd
        set b_i to 1
    if D is even
        set b_i to 0
    i++
    D = D/2
```

Example: Converting 105

idea:		example:	
	D	D = 105	b_0 = 1
	D = D/2	D = 52	b_1 = 0
	D = D/2	D = 26	b_2 = 0
	D = D/2	D = 13	b_3 = 1
	D = D/2	D = 6	b_4 = 0
	D = D/2	D = 3	b_5 = 1
	D = D/2	D = 1	b_6 = 1
	D = 0 (done)	D = 0	b_7 = 0

105 = 01101001

Method 2

- $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$
- To convert <u>105</u>:
  - Find largest power of two that's less than 105 (64)
  - Subtract 64 (105 – 64 = <u>41</u>), put a 1 in d_6
  - Subtract 32 (41 – 32 = <u>9</u>), put a 1 in d_5
  - Skip 16, it's larger than 9, put a 0 in d_4
  - Subtract 8 (9 – 8 = <u>1</u>), put a 1 in d_3
  - Skip 4 and 2, put a 0 in d_2 and d_1
  - Subtract 1 (1 – 1 = <u>0</u>), put a 1 in d_0 (Done)

1	1	0	1	0	0	1
d_6	d_5	d_4	d_3	d_2	d_1	d_0

What is the value of 357 in binary?

A. 101100011
B. 101100101
C. 101101001
D. 101110101
E. 110100101

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$,

$2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$

So far: Unsigned Integers

- With N bits, we can represent values: 0 to 2^n-1

- We can always add 0's to the front of a number without changing it:

10110 = 010110 = 00010110 = 0000010110

- `1 byte: char, unsigned char`
- `2 bytes: short, unsigned short`
- `4 bytes: int, unsigned int, float`
- `8 bytes: long long, unsigned long long, double`
- `4 or 8 bytes: long, unsigned long`

Coming up next…

- How do we store *signed* integers?
- How do we perform arithmetic on binary values?
- What are the limits on what we can store in a certain number of bits?

Aside: Floating Point Representation

1 bit for sign
8 bits for exponent
23 bits for precision

sign | exponent | fraction |

value = $(-1)^{sign} * 1.fraction * 2^{(exponent-127)}$

Let's plug in a value and try it out:

```
0x40ac49ba: 0 10000001   01011000100100110111010
      sign = 0 exp = 129   fraction = 2902458
```

$= 1*1.2902458*2^2 = 5.16098$

I don’t expect you to memorize this!