CS46 practice problems 10

These practice problems are an opportunity for discussion and trying many different solutions. It is not counted towards your grade, and you do not have to submit your solutions. The purpose of these problems is to get more comfortable with reasoning and writing proofs about decidability, recognizability, and co-recognizability.

If you are stumped or looking for guidance, ask.

1. Consider the language $L = \{\langle M, w \rangle \mid M$ is a single-tape TM that never modifies the portion of the tape that contains the original input $w\}$.

   (a) Show that $L$ is co-Turing-recognizable, by briefly describing the elements of $\overline{L}$ and then describing a recognizer for $\overline{L}$.

   (b) Is $L$ decidable? Prove your answer.

   Note that if you can show that $L$ is Turing-recognizable, then you can apply Theorem 4.22 and part (a) to show $L$ is decidable.

2. For each of the following languages, review if the language is decidable, Turing-recognizable, co-Turing-recognizable, or none of these. $A_{\text{DFA}}, A_{\text{CFG}}, A_{\text{TM}}, E_{\text{DFA}}, E_{\text{CFG}}, E_{\text{TM}}, \text{ALL}_{\text{DFA}}, \text{ALL}_{\text{CFG}}, \text{ALL}_{\text{TM}}, \text{EQ}_{\text{DFA}}, \text{EQ}_{\text{CFG}}, \text{EQ}_{\text{TM}}$.

3. Consider the language of Turing machines which only accept strings consisting of $a$s and $b$s:

   $\text{TargetGPA}_{\text{TM}} = \{\langle M \rangle \mid M$ is a Turing machine and $L(M) \subseteq L((a \cup b)^*)\}$

   Is this language regular? context-free? decidable? recognizable? co-recognizable?

   You may consider these parts in any order. (Some orders will be more helpful than others.) Support your answer for each part with a proof!

4. Consider the language of deciders:

   $\text{DECIDER}_{\text{TM}} = \{\langle M \rangle \mid M$ is a decider\}

   (a) Recall that $L_{\text{TM}} = \{\langle M \rangle \mid M$ is a Turing machine\}, so $\text{DECIDER}_{\text{TM}} \subseteq L_{\text{TM}}$. Does Rice’s Theorem apply to $\text{DECIDER}_{\text{TM}}$?

   (b) Show that $\text{DECIDER}_{\text{TM}}$ is undecidable.

   (c) Show that $\text{DECIDER}_{\text{TM}}$ is both not recognizable and not co-recognizable.

ONLY IF you finish problems 1-4, look at problem 5. The busy beaver problem is interesting, and famous, and challenging. It is also much, much more complicated than any problem I would ever ask on a homework or exam. It’s here for your intellectual enjoyment!