CS46 Homework 3

This homework is due at 11:59PM on Sunday, February 6. Write your solution using \LaTeX. Submit this homework using github. This is a 10 point homework.

This is an individual homework. It’s ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you must cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main learning goal of this homework is to develop the skills to design, understand, and analyze DFAs, and to think about the class of regular languages in general.

Part 1 — These problems should be completed on Automata Tutor. You are allowed three attempts at each problem. I recommend that you first try to solve the problems on paper, then use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.

2. Construct a DFA for the language $\{\varepsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.

3. Construct a DFA for the language $\{w \mid w$ is either $a$ or $b\}$ over alphabet $\Sigma = \{a, b\}$.

4. Construct a DFA for the language $\{w \mid w$ is any string except $a$ or $b\}$ over alphabet $\Sigma = \{a, b\}$.

5. Construct a DFA for the language $\{w \mid w$ contains at least three $1$s\} over alphabet $\Sigma = \{0, 1\}$.

6. Construct a DFA for the language $\{w \mid$ every $a$ in $w$ is immediately followed by a $b\}$ over alphabet $\Sigma = \{a, b\}$.

7. Construct a DFA for the language $\{w \mid b$ occurs $n$ times in $w$, where $n$ is divisible by $3\}$ over alphabet $\Sigma = \{a, b\}$.

8. Construct a DFA for the language $\{w \mid$ length of $w \leq 5\}$ over alphabet $\Sigma = \{a, b\}$.

9. Construct a DFA for the language $\{w \mid w$ contains at least two $0$s and at most one $1\}$ over alphabet $\Sigma = \{0, 1\}$.

10. Construct a DFA for the language $L = \{w \mid$ every odd position of $w = w_1w_2w_3\ldots w_n$ is a $1\}$ over the alphabet $\Sigma = \{0, 1\}$.

11. Construct a DFA for the language $L = \{w \mid w$ is any non-empty string$\}$ over the alphabet $\Sigma = \{0, 1\}$.

12. Construct a DFA for the language $L = \{w \mid w$ begins and ends with the same symbol$\}$ over the alphabet $\Sigma = \{0, 1\}$. This language includes the empty string.

\footnote{If you want to use late days on this assignment, you will need to submit solutions to these problems via github. The automatatutor site has only one deadline.}
13. **Extra credit.** Construct a DFA for the language \( L = \{ w \mid w \text{ is a binary number equal to } 2 \text{ mod } 5 \} \) over alphabet \( \Sigma = \{0, 1\} \). (So \( 0 \notin L \), \( 10 \in L \), \( 100 \notin L \), etc.)

**Part 2** — These problems should be typeset in \LaTeX{} and submitted using github.

14. Let \( \Sigma = \{a, b, c, \ldots, z\} \). For any language \( A \subseteq \Sigma^* \), let the **contrary** of \( A \) be defined as:

\[
\text{contrary}(A) = \{ \text{anti}w \mid w \in A \}
\]

For example, if \( A = \{\text{unicorn}, \text{pony}, \text{tricycle}\} \), then

\[
\text{contrary}(A) = \{\text{ant unicorn}, \text{ant pony}, \text{ant tricycle}\}
\]

Prove that the class of regular languages is closed under the “contrary” operator. (That is, prove that if \( A \) is regular, then \( \text{contrary}(A) \) is regular. You should describe how to construct a machine that recognizes \( \text{contrary}(A) \), define all elements of your machine \( M = (Q, \Sigma, \delta, q_0, F) \), and argue why this machine recognizes \( \text{contrary}(A) \).

15. Consider the language \( C = \text{op}(A, B) \) where “op” is some operation that regular languages are closed under. Suppose we know the following about \( A \) and \( C \). What, if anything, can we conclude about \( B \)?

(You should support your answer with a brief explanation. Even though we have not yet seen any specific languages that are not regular, you can approach this problem using just the definition of “regular language” and “closed”.)

(a) \( A \) is regular and \( C \) is regular.
(b) \( A \) is regular and \( C \) is not regular.
(c) \( A \) is not regular and \( C \) is regular.
(d) \( A \) is not regular and \( C \) is not regular.

16. **(extra credit)** For languages \( A \) and \( B \), let the **perfect shuffle** of \( A \) and \( B \) be the language:

\[
\{ w \mid w = x_1y_1x_2y_2 \cdots x_ky_k \text{ where } x_1, \ldots, x_k \in A \text{ and } y_1, \ldots, y_k \in B \text{ and each } x_i, y_i \in \Sigma \}
\]

Prove that the class of regular languages is closed under perfect shuffle.