

CS46 practice problems 7

These practice problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions**. The purpose of these problems is to get more comfortable with reasoning and writing about Turing machines, decidability, and recognizability.

If you are stumped or looking for guidance, some of these problems are in the “selected solutions” portion of the textbook. Others we will discuss in lab.

1. Give an **implementation-level** description of a Turing machine that, when given an input $w \in \{a, b\}^*$, shifts w one square to the right, resulting in $\sqcup w$ on the input tape.
2. Give an **implementation-level** description of a Turing machine that decides the following language over the alphabet $\{0, 1\}$:

$$\{w \mid w \text{ contains an equal number of 0s and 1s}\}$$

3. Show that ALL_{DFA} is decidable, where ALL_{DFA} is defined as:

$$ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$$

(Hint: recall which languages are decidable from lecture last week, and which operations we know how to build DFAs for, waaaaay back when we proved things like “regular languages are closed under intersection”.)

4. Show that $INFINITE_{DFA}$ is decidable, where $INFINITE_{DFA}$ is defined as:

$$INFINITE_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is an infinite language}\}$$

5. Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $A = \{\langle M_1 \rangle, \langle M_2 \rangle, \dots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A .

(Hint: you may find it helpful to consider an enumerator for A .)