For clarity, consider a language $L$.

1. Is $L$ decidable?
2. How would you design a TM to decide $L$?

Design it. Give the algorithm.

These questions are different!

If you can give a TM in QZ, then Q1 is YES. Sometimes we can prove Q1 is YES even though we don't have enough info to answer Q2.

Define

$$A_{TM} = \{ <M, w> : M \text{ is a TM which accepts string } w \}$$

$A_{TM}$ is not decidable

$A_{TM}$ is recognizable

recognizable \& $A_{TM}$

decidable
1 & M is a TM, define $L(M) = \{ w \mid M \text{ accepts } w \}$. This is the language recognized by $M$.

<table>
<thead>
<tr>
<th>list of all TMs</th>
<th>all Turing-recognizable languages</th>
<th>all co-recognizable languages</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt;M_1&gt;$</td>
<td>$L(M_1)$</td>
<td>$L(M_1)$</td>
</tr>
<tr>
<td>$&lt;M_2&gt;$</td>
<td>$L(M_2)$</td>
<td></td>
</tr>
<tr>
<td>$&lt;M_3&gt;$</td>
<td>$L(M_3)$</td>
<td>$L(M_3)$</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Countable ✓ (might contain duplicates)

Claim: $\overline{\text{ATM}}$ is not decidable.

Proof: (by contradiction)

Suppose $M$ decides $\overline{\text{ATM}}$. 
\( N = \"\text{On input } (R, w)\:\) 
1. Run \( M \) on \( (R, w) \).
2. Do the opposite.\"

\( N \) is a decider for \( A_{TM} \).

\( \Rightarrow \) \( \Leftarrow \) We know \( A_{TM} \) is undecidable!