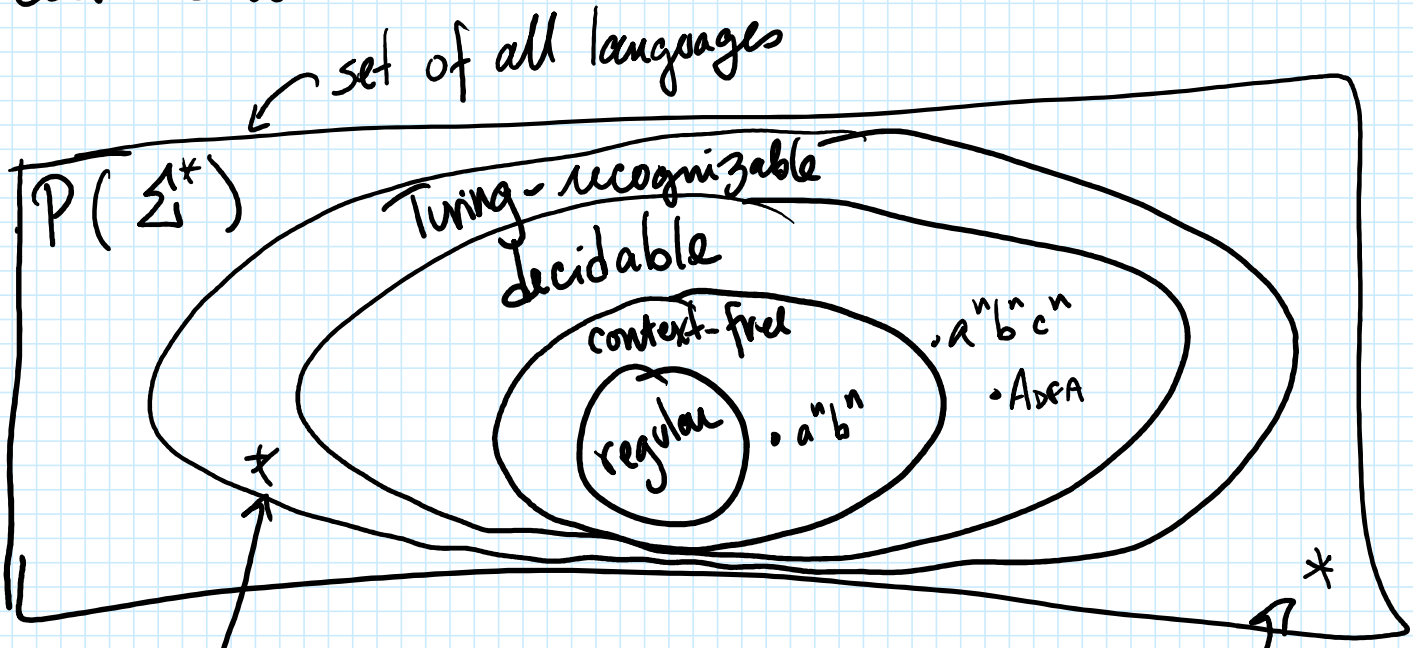
# Decidability

Monday, March 23, 2020 9:01

## Sets of languages we've seen:

- regular
- context-free
- Turing-recognizable
- decidable




Is there any language that is recognizable but not decidable?

Is there any unrecognizable lang?

$$A_{DFA} = \{ \langle D, w \rangle \mid \begin{array}{l} D \text{ is a DFA} \\ w \text{ is a string} \\ D \text{ accepts } w \end{array} \}$$

**Claim**: $A_{DFA}$ is decidable.

**Pf**: (direct)

Let's build a TM $M$ to decide $A_{DFA}$.

$M$ = "On input $x$:

1. If $x$ is not a properly-formatted pair $\langle D, w \rangle$ where $D$ is a DFA, then reject.
2. Simulate $D$ on input $w$

   (start in $D$'s $q_0$ state
   for each character of $w$:
   move to a new state of $D$
   according to $D$'s transitions)

3. If $D$ is in an accept state, accept.
   Else, reject."

**Need to check**:

- is $M$ a decider? ✓
- is $L(M) = A_{DFA}$? ✓

  if $D$ accepts $w$, then $M$ accepts $\langle D, w \rangle$.

  if $D$ rejects $w$, then $M$ rejects $\langle D, w \rangle$.

  or is not a (DFA, string) pair, then $M$ rejects.

Rewrite $M$ in a more compact way:

Rewrite $M$ in a more compact way:

$M$ = "On input $\langle D, w \rangle$ where $D$ is a DFA & $w$ a string:

1. Simulate $D$ on $w$. // This line is hiding a LOT of detail.
2. If $D$ accepted $w$, accept.
   Else, reject."