Sets of languages we’ve seen:
- regular
- context-free
- Turing-recognizable
- decidable

set of all languages

\( P(\Sigma^*) \)

Turing-recognizable

decidable

context-free

regular

Is there any language that is recognizable but not decidable?

Is there any unrecognizable language?

\[ A_{\text{DFA}} = \left\{ \langle D, w \rangle : D \text{ is a DFA} \text{ and } w \text{ is a string} \text{ that } D \text{ accepts} \right\} \]
**Claim:** $A_{DFA}$ is decidable.

**Pf:** (direct)

Let's build a TM $M$ to decide $A_{DFA}$.

$M =$ "On input $x$:

1. If $x$ is not a properly-formatted pair $<D,w>$ where $D$ is a DFA, then reject.

2. Simulate $D$ on input $w$
   - (stand in $D$'s go state for each character of $w$: move to a new state of $D$ according to $D$'s transitions)

3. If $D$ is in an accept state, accept. Else, reject."

**Need to check:**

- is $M$ a decider? ✓
- is $L(M) = A_{DFA}$? ✓

if $D$ accepts $w$, then $M$ accepts $<D,w>$.
if $D$ rejects $w$, then $M$ rejects $<D,w>$.
or is not a (DFA, string) pair, then $M$ rejects.

Rewrite $M$ in a more compact way:
Rewrite $M$ in a more compact way:

$M = \"\text{On input } \langle D, w \rangle \text{ where } D \text{ is a DFA & } w \text{ a string:}\n\begin{align*}
1. & \text{Simulate } D \text{ on } w. \quad \text{\(//\) This line is hiding a LOT of detail.} \\
2. & \text{If } D \text{ accepted } w, \text{ accept.} \\
    & \text{Else, reject.}\"\end{align*}$