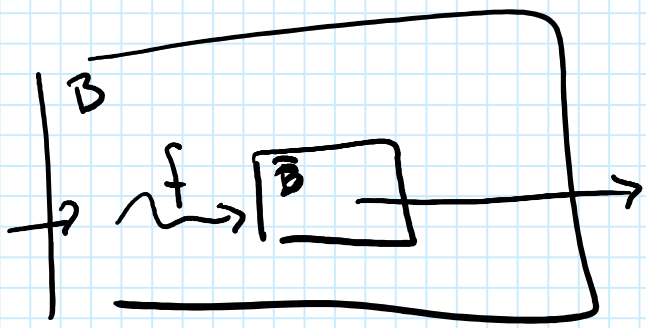
W10L3 one more day of $\leq_M$ practice

Friday, April 10, 2020 9:14

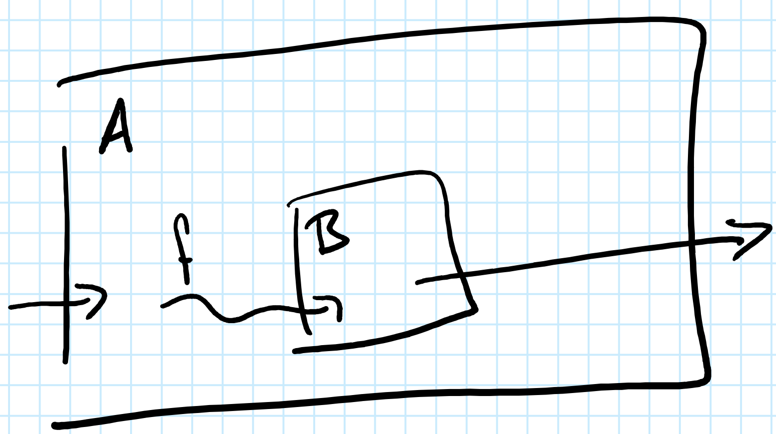
Clicker Q1:

$B \leq_m \overline{B}$



We know if B is decidable so is $\overline{B}$. ← generally true

Clicker Q2:



Thm (previously)

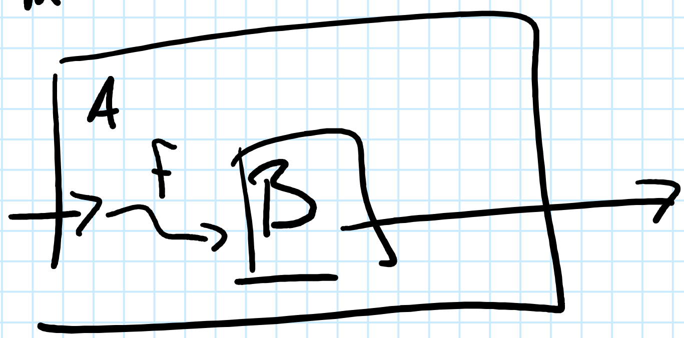
A language L is decidable

iff

L is recognizable & co-recognizable.

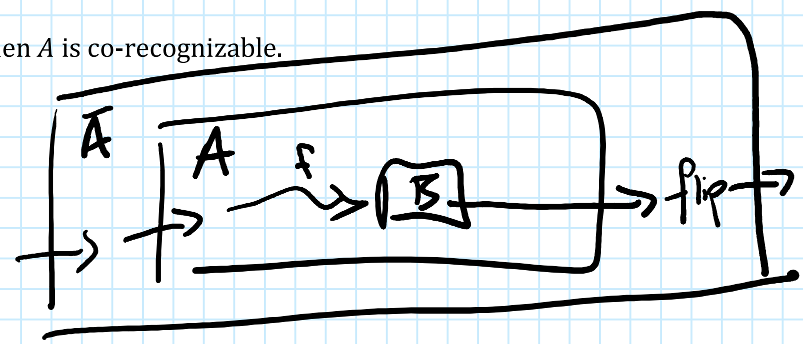
L is recognizable & co-recognizable

If $A \le_m B$:

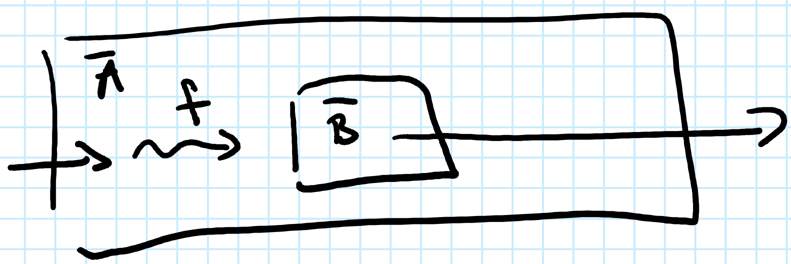


Thm: $A \le_m B$ iff $\overline{A} \le_m \overline{B}$.

Conclusions we've drawn from $A \le_m B$:
- If B is decidable, then A is decidable.
- If B is recognizable, then A is recognizable.
- $\overline{A} \le_m \overline{B}$
- If B is co-recognizable, then A is co-recognizable.



$\overline{A} \le_m \overline{B}$



Rice's Thm

$L_{TM} = \{ \langle M \rangle \mid M \text{ is a Turing machine} \}$

Def: A <u>PROPERTY</u> $P \subseteq L_{TM}$ must satisfy that whenever $L(M_1) = L(M_2)$

then either ① $\langle M_1 \rangle$ and $\langle M_2 \rangle \in P$

② $\langle M_1 \rangle$ and $\langle M_2 \rangle \notin P$.

<u>Rice's Thm</u>: If P is a property and $P \neq \emptyset$ and $P \neq L_{TM}$, then P is undecidable.

This thm is ONLY useful to show that languages of the form $\{ \langle M \rangle \mid M$ is a TM and ________ $\}$ are undecidable.

<u>example</u>:

$E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$

In order to apply Rice's Theorem, we need to check:

- E_{TM} is not $\emptyset$: it contains at least one machine, for example "on input x: always reject"
- E_{TM} is not L_{TM}: it does not contain at least one machine, for example "on input x: always accept"
- E_{TM} is a property: If $L(M_1) = L(M_2)$ then either they both equal $\emptyset$ (so $< M_1 >, < M_2 > \in E_{TM}$, or they both are something else, and $< M_1 >, < M_2 >$ not $\in E_{TM}$.

Thus we can conclude by Rice's theorem that E_{TM} is not decidable. □