Question 1:

\[ B \leq_M \overline{B} \]

We know if \( B \) is decidable so is \( \overline{B} \). \(-\text{generally true}\)

Question 2:

Theorem (previously):

A language \( L \) is decidable if and only if \( L \) is recognizable and co-recognizable.
Conclusions we’ve drawn from \( A \leq_m B \):
- If \( B \) is decidable, then \( A \) is decidable.
- If \( B \) is recognizable, then \( A \) is recognizable.
- \( \overline{A} \leq_m \overline{B} \)
- If \( B \) is co-recognizable, then \( A \) is co-recognizable.

Rice's Thm

\[
L_{TM} = \{ <M> | M \text{ is a Turing machine} \}
\]

Def: A property \( P \) \( \leq \) \( L_{TM} \) must satisfy that whenever \( L(M_1) = L(M_2) \)
then either
\(\text{(i)}\) \(<M_1>\) and \(<M_2>\) \(\in\ D\)
\(\text{(ii)}\) \(<M_1>\) and \(<M_2>\) \(\notin\ D\).

Rice's Thm.: If \(D\) is a property and
\(\neg\emptyset\) and \(D \neq L_{TM}\), then \(D\) is undecidable.

This thm is ONLY useful to show that languages of the form \(\exists <M> | M\text{ is a TM and ______}\) are undecidable.

\(E_{TM} = \{<M> | M\text{ is a TM and } L(M) = \emptyset\}\)

In order to apply Rice's Theorem, we need to check:
- \(E_{TM}\) is not \(\emptyset\): it contains at least one machine, for example "on input x: always reject"
- \(E_{TM}\) is not \(L_{TM}\): it does not contain at least one machine, for example "on input x: always accept"
- \(E_{TM}\) is a property: If \(L(M_1) = L(M_2)\) then either they both equal \(\emptyset\) (so \(<M_1>, <M_2> \in E_{TM}\), or they both are something else, and \(<M_1>, <M_2> \notin E_{TM}\).

Thus we can conclude by Rice's theorem that \(E_{TM}\) is not decidable. \(\square\)