W10L2 reductions 'n' reductions 'n' reductions

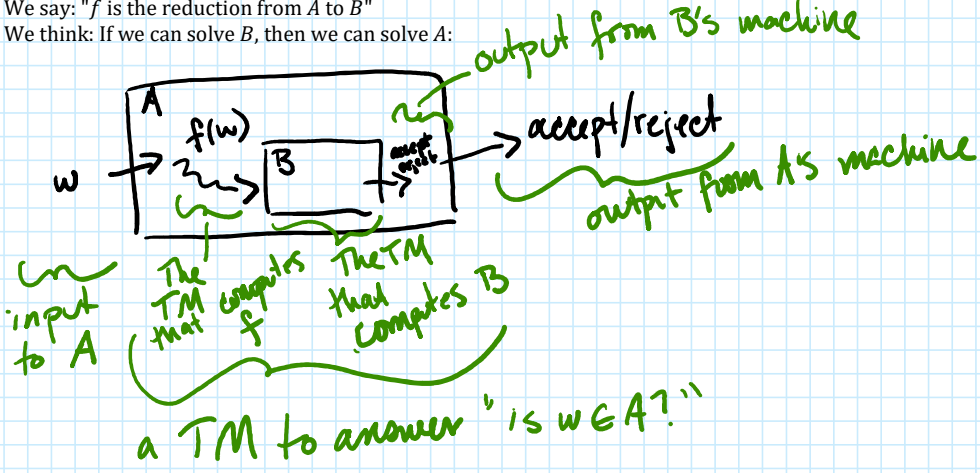
Wednesday, April 8, 2020 9:13

Announcements:
- quiz this week! TRY TO TAKE IT PLEASE
- pick a homework 8 partner

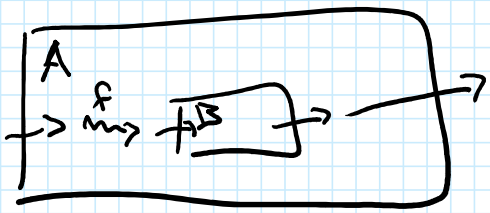
Def: A function $f: \Sigma^* \to \Sigma^*$ is **computable** iff there is some Turing machine M which on every input w eventually halts with just $f(w)$ on the tape.

Def: For two languages A and B, we will say that A is **mapping-reducible** to B if there is a computable function f such that $\forall w, w \in A \Leftrightarrow f(w) \in B$.

We write: $A \leq_m B$
We say: "f is the reduction from A to B"
We think: If we can solve B, then we can solve A:



Clicker Q1: $A \leq_m B$ & A is decidable.



B might be decidable, or not.

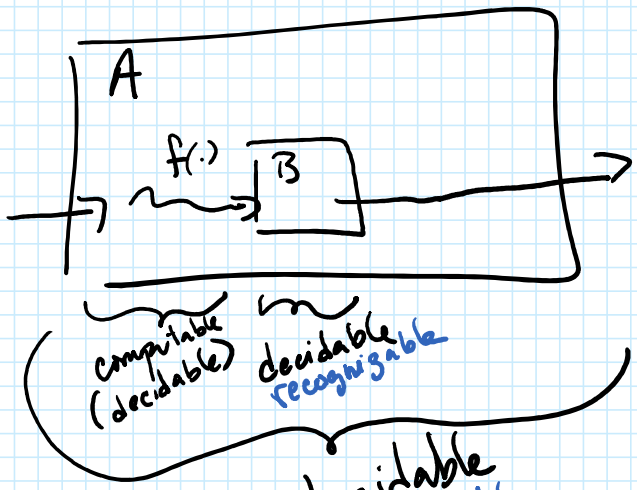
ex: $B = E_{TM}$
$A = L_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine}\}$

$$f(w) = \begin{cases} \langle M_R \rangle = \text{"On input } x\text{: reject"}, & \text{if } w \text{ was a TM encoding} \\ \langle M_A \rangle = \text{"On input } x\text{: accept"}, & \text{if } w \text{ was not a TM encoding} \end{cases}$$

Need to check:

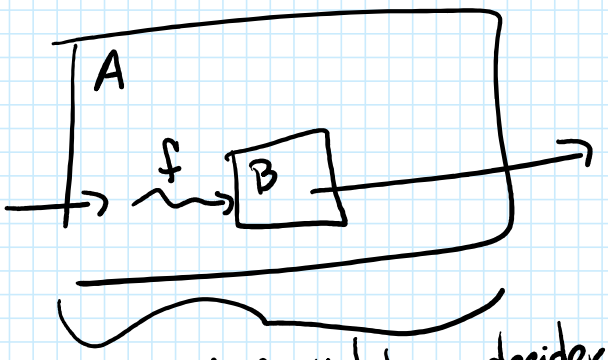
- f is a computable function:
  A TM computing $f(w)$ can just
  (1) check the formatting of w, and
  (2) if w encoded a TM, erase the tape and write $< M_R >$ on the tape, and halt;
  (3) if w did not encode a TM, erase the tape and write $< M_A >$ on the tape and halt.
- If $w \in A = L_{TM}$ then $f(w) \in B = E_{TM}$.
- If w not $\in A$ then $f(w)$ is not $\in B$.

Clicker Q2: $A \leq_m B$ & B is decidable
recognizable



A is decidable
recognizable

Clicker Q3: $A \leq_m B$ and A is undecidable
unrecognizable



This TM cannot be a decider!
recognizer

If B were decidable, then A would be decidable.
recognizable recognizable
This contradicts the fact that A is undecidable!
unrecognizable
So, B must be undecidable.
unrecognizable