CS46 Homework 9

This homework is due at 10pm on Sunday, April 26. Submit on github as a file called **hw9.tex**.

For this homework, you will work with a partner or alone. It's ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership's write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy. or ask me any questions you have about this policy.

1. Use the definition of big-O to prove that:
   (a) $2^n \in O(5^n)$.
   (b) $n^2 \log n \in O(n^3)$.
   (c) $\frac{1}{7} \cdot 3^n \notin O(n^2)$.
2. A **triangle** in a graph is three nodes that are all connected to each other by edges. Show that TRIANGLE $\in$ P, where

$$\text{TRIANGLE} = \{\langle G \rangle \mid G \text{ contains a triangle}\}$$

3. **Closure properties.**
   (a) Prove that P is closed under concatenation.
   (b) Prove that P is closed under complement.
   (c) Prove that NP is closed under union.
   (d) Prove that NP is closed under concatenation.
4. Show that if coNP $\neq$ NP, then P $\neq$ NP.
5. **(extra credit)** Does coNP = NP? Support your answer with a proof.