CS46 Homework 7

This homework is due at 10pm on Sunday, April 5. This is a **7 point** homework.

For this homework, you will work with a partner or alone. It's ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership's write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about Turing machines, decidable languages, and how to use (possibly *in combination*) the many proof tools and facts we have gathered over the past several weeks. As always, we shall continue to monotonically improve our proof-writing, clarity, and organization skills.

1. Prove that the following language is decidable:

$$\{\langle M \rangle \mid M \text{ is a DFA and } \forall w, \text{ if } w \in L(M) \text{ then } w^R \in L(M)\}$$

   Be thorough! For example, if you build a decider, then you need to prove that it *is* a decider, and that it decides this language exactly.

2. Show that every infinite Turing-recognizable language has an infinite decidable subset.

3. **(extra credit)** Let $B = \{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$ be a Turing-recognizable language consisting of Turing machine descriptions. Show that there is a decidable language C consisting of Turing machine descriptions such that every machine described in B has an equivalent machine in C, and every machine described in C has an equivalent machine in B.

4. **(extra credit)** A_{TM} is a language consisting of descriptions of Turing machines, and it is Turing-recognizable. Why does the previous question not imply that A_{TM} is decidable?