CS46 Homework 4

This homework is due at 10pm on Sunday, February 23.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership's write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to work with our tools for proving that languages are/are not regular, and designing CFGs. As always, we shall continue to monotonically improve our proof-writing, clarity, and organization skills.

1. Give a context-gree grammar that generates the language

$$\{a^i b^j c^k \mid i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$$

   You do not have to give a proof of correctness, but you should think about what would be required to write a proof. This will help you debug your grammar. (Note: we saw a PDA for this language in lecture.)

2. Consider the class of context-free languages.

   (a) Using constructive proofs, build context-free grammars that demonstrate the class of context-free languages is closed under the regular operations of union, concatenation, and Kleene star. (You should build three separate grammars, and argue that each is correct.)

   (b) Theorem 1.25 proves that the class of regular languages is closed under intersection. Technically, it proves closure under union, but as the footnote in step 5 notes, a slight tweak makes this proof work for intersection, too. Can a similar technique be applied to pushdown automata to show the class of context-free languages is closed under intersection? Explain your answer briefly, but you do not need to give a full proof/counterargument.

3. Consider the following languages. For each, is the language regular? Support your claim with a proof.

   (a) $L_1 = \{a^k u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (b) $L_2 = \{a^k b u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (c) $L_3 = \{a^n b^m a^m b^n \mid m, n \geq 0\}$ where $\Sigma = \{a, b\}$.

   (d) $L_4 = \{a^{m-n} \mid \frac{m}{n} = 3\}$ where $\Sigma = \{a, b\}$.

   (e) $L_5 = \{w \mid w \text{ is not a palindrome}\}$ where $\Sigma = \{a, b\}$.

   (f) $L_6 = \{w \mid w = x_1 \# x_2 \# \cdots \# x_k \text{ for } k \geq 0, \text{ each } x_i \in L(a^*), \text{ and } x_i \neq x_j \text{ for } i \neq j\}$, where $\Sigma = \{a, \#\}$.

4. **(extra credit)** Consider the grammar G with rules: $\left\{ \begin{array}{ll} S & \rightarrow AS \mid \varepsilon \\ A & \rightarrow 0A \mid A1 \mid \varepsilon \end{array} \right.$

(a) Show that G is ambiguous.

(b) Give an equivalent grammar to G which is not ambiguous. (No proof is required, but you should explain why it's not ambiguous and why it generates exactly the same strings as G.)

5. **(dangerously extra credit)** Consider the languages from lab this week (practice problems 4) and from this homework. For any of those languages that are context-free and not regular, give a CFG or PDA for that language. (Warning: some of the languages might not even be context-free, so... use your time wisely. We are still learning the tools to deal with all of these.)