CS46 Homework 2

This homework is due at 10:00PM on Sunday, February 9. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** homework.

This is an individual homework. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to develop the skills to design, understand, and analyze DFAs and regular languages.

Part 1 — These problems should be completed[1] on Automata Tutor. You are allowed **three attempts** at each problem. I recommend that you *first* try to solve the problems on paper, *then* use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.
2. Construct a DFA for the language $\{\varepsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.
3. Construct a DFA for the language $\{w \mid w \text{ is either } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
4. Construct a DFA for the language $\{w \mid w \text{ is any string except } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
5. Construct a DFA for the language $\{w \mid w \text{ contains at least three 1s}\}$ over alphabet $\Sigma = \{0, 1\}$.
6. Construct a DFA for the language $\{w \mid \text{every } a \text{ in } w \text{ is immediately followed by a } b\}$ over alphabet $\Sigma = \{a, b\}$.
7. Construct a DFA for the language $\{w \mid b \text{ occurs } n \text{ times in } w, \text{ where } n \text{ is divisible by } 3\}$ over alphabet $\Sigma = \{a, b\}$.
8. Construct a DFA for the language $\{w \mid \text{ length of } w \leq 5\}$ over alphabet $\Sigma = \{a, b\}$.
9. Construct a DFA for the language $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$ over alphabet $\Sigma = \{0, 1\}$.
10. Construct a DFA for the language $L = \{w \mid \text{ every odd position of } w = w_1 w_2 w_3 \dots w_n \text{ is a } 1\}$ over the alphabet $\Sigma = \{0, 1\}$.
11. Construct a DFA for the language $L = \{w \mid w \text{ is any non-empty string}\}$ over the alphabet $\Sigma = \{0, 1\}$.
12. Construct a DFA for the language $L = \{w \mid w \text{ begins and ends with the same symbol}\}$ over the alphabet $\Sigma = \{0, 1\}$. This language includes the empty string.

[1] If you want to use late days on this assignment, you will need to submit solutions to these problems via github. The automatatutor site has only one deadline.

13. Construct an NFA for the language $L = \{w \mid w \text{ contains an odd number of 1s or exactly two 0s}\}$ over the alphabet $\Sigma = \{0, 1\}$. Your NFA should have no more than six states.

14. Construct an NFA with two states for the language $\{0\}$ over the alphabet $\{0, 1\}$.

15. **Extra credit.** Construct a DFA for the language $L = \{w \mid w \text{ is a binary number equal to 2 mod 5}\}$ over alphabet $\Sigma = \{0, 1\}$. (So $0 \notin L$, $10 \in L$, $100 \notin L$, etc.)

Part 2 — These problems should be typeset in LaTeXand submitted using **github**.

16. Write a concise English description of the language recognized by DFA M_1.

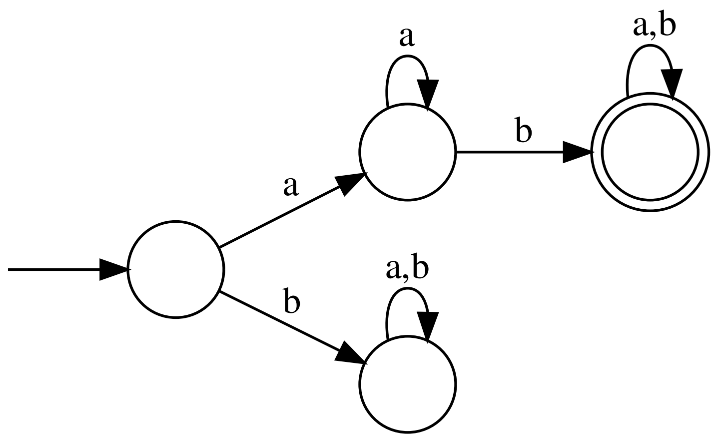

Figure 1: DFA M_1

17. Write a concise English description of the language recognized by DFA M_2.

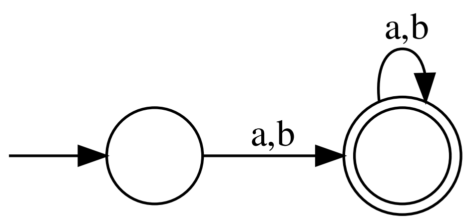

Figure 2: DFA M_2

18. Write a concise English description of the language recognized by DFA M_3.

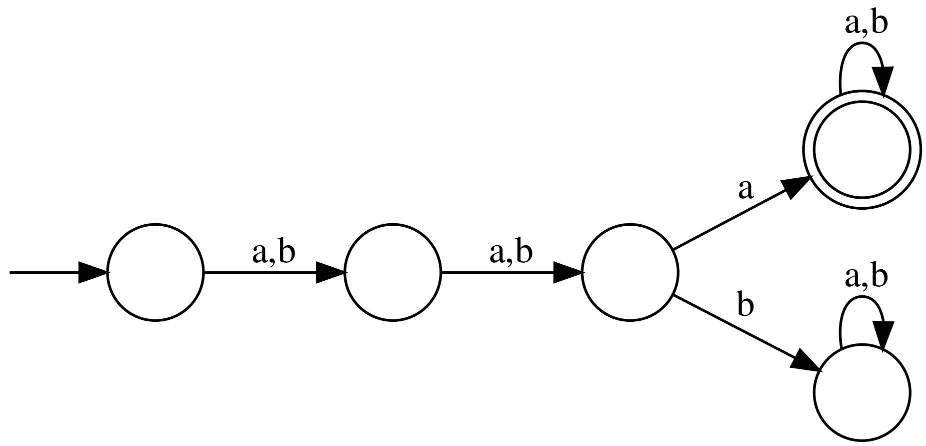

Figure 3: DFA M_3

19. Let $\Sigma = \{a, b, c, \ldots, z\}$. For any language $A \subseteq \Sigma^*$, let the **contrary** of A be defined as:

$$\text{contrary}(A) = \{antiw \mid w \in A\}$$

For example, if $A = \{unicorn, pony, tricycle\}$, then

$$\text{contrary}(A) = \{antiunicorn, antipony, antitricycle\}$$

Prove that the class of regular languages is closed under the "contrary" operator. (That is, prove that if A is regular, then contrary(A) is regular. You should describe how to construct a machine that recognizes contrary(A), define all elements of your machine $M = (Q, \Sigma, \delta, q_0, F)$, and argue why this machine recognizes contrary(A).)

20. (**extra credit**) For languages A and B, let the **perfect shuffle** of A and B be the language:

$$\{w \mid w = \sigma_1\gamma_1\sigma_2\gamma_2 \cdots \sigma_k\gamma_k \text{ where } \sigma_1 \cdots \sigma_k \in A \text{ and } \gamma_1 \cdots \gamma_k \in B \text{ and each } \sigma_i, \gamma_i \in \Sigma\}$$

Prove that the class of regular languages is closed under perfect shuffle.