CS46 Homework 1

This homework is due at 10:00pm on Sunday, February 2. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** homework.

This is an individual homework. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to cement your understanding of mathematical concepts that we will be using throughout the semester.

1. Give the cardinality of the following sets:
   (a) $\{a\}$
   (b) {☺, $\emptyset$}
   (c) $\{a, b, c, \{b, \emptyset,$ 😄$\}\}$
   (d) $\emptyset$
   (e) $\{\{\emptyset\}, \{a, \emptyset,$ 😐$\}\}$

2. Set operations.
   (a) If A is a set of size k, how many elements are in the powerset of A? Explain your answer.
   (b) How many bit strings of length n are there? Explain your answer.
   (c) Let $A = \{a, b, c\}$ and $B = \{$🥷, 🐱, 😇, $a\}$. What are the sets $A \cap B$, $A \cup B$, and $A \times B$? Formally describe them by listing their elements.
   (d) List the elements of the powerset of $A = \{$😉, $\emptyset$, 😕$\}$.
   (e) Let $\Sigma = \{a, b\}$. List all strings over Σ of length 3 in short lexicographic order.

3. For each of these requirements, describe a relation (either formally or in English sentences) which is:
   (a) symmetric but not transitive.
   (b) transitive and reflexive but not symmetric.
   (c) reflexive, symmetric, and transitive.

   You can pick the domain of each relation, but make sure to specify it in your write-up.

4. Let Σ be an alphabet (a set of letters). We define Σ^* as the set of all strings using letters from Σ. Let $\mathcal{C}$ be a collection of sets which are all subsets of Σ^*. We are given that $\Sigma^* \in \mathcal{C}$.

   Assume that $\mathcal{C}$ is closed under the operation set difference. (So if $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \backslash B \in \mathcal{C}$.)

   Using direct proof, show that:

(a) If $A \in \mathcal{C}$, then $\bar{A} \in \mathcal{C}$. ($\mathcal{C}$ is closed under complement.)

(b) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cap B \in \mathcal{C}$. ($\mathcal{C}$ is closed under intersection.)

(c) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cup B \in \mathcal{C}$. ($\mathcal{C}$ is closed under union.)

5. **(extra credit)** Formally prove that $n^2 + n$ is divisible by 2 for all $n \in \mathbb{N}$.