CS46 Homework 9

This homework is due at 11:59pm on Tuesday, April 11. For this homework, you will work with a partner. It’s ok to discuss approaches at a high level, but most of your discussions should just be with your partner. The only exception to this rule is work you’ve done with another student while in lab. In this case, note who you’ve worked with and what parts were solved during lab.

Your partnership’s write-up should be your own: do not share it, and do not read other people’s write-ups. Please refer to the course webpage or ask me any questions you have about this policy.

0. Before final submission, make sure to fill out the README file.

1. Use the definition of big-O to prove that:
   (a) $2^n = O(5^n)$.
   (b) $n^2 \log n = O(n^3)$.
   (c) $\frac{1}{2} \cdot 3^n \neq O(n^2)$.

2. Closure properties.
   (a) Prove that \(P\) is closed under concatenation.
   (b) Prove that \(P\) is closed under complement.
   (c) Prove that \(NP\) is closed under union.
   (d) Prove that \(NP\) is closed under concatenation.

3. A Boolean formula is in conjunctive normal form (CNF) if it is written as the conjunction of clauses, for example:

   $$(x_1 \lor x_2) \land (\overline{x}_2 \lor \overline{x}_3 \lor x_4) \land (x_5 \lor \overline{x}_1 \lor x_6) \land (x_3)$$

   (Recall that a literal is a Boolean variable or a negated Boolean variable, like \(x\) or \(\overline{x}\), and a clause is a disjunction of literals, like \(x \lor y \lor \overline{z}\). The symbol “\(\lor\)” means “or”; the symbol “\(\land\)” means “and”.)

   (a) (Sipser 7.24a) Define the language:

   $$L = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable CNF formula where each variable appears at most twice} \}$$

   Show that \(L \in P\).

   (b) (Sipser 7.43) For a CNF formula \(\phi\) with \(m\) variables and \(c\) clauses, show you can construct in polynomial time an NFA with \(O(cm)\) states that accepts all nonsatisfying assignments, represented as binary strings of length \(m\).

   (This implies that if \(P \neq NP\), then NFAs cannot be minimized in polynomial time.)