CS46 Homework 2

This homework is due at 9AM on Monday, January 30. Part 1 should be submitted on [Automata Tutor] part 2 should be written using LaTeX and submitted using [github]. This is an individual homework. It’s ok to discuss approaches at a high level, but your write-up should be your own: do not share it, and do not read other people’s write-ups. Please refer to the course webpage or ask me any questions you have about this policy.

0. Before final submission, make sure to fill out the README file.

**Part 1** — These problems should be completed and submitted on [Automata Tutor]. In order to do this you should:

- Create a login on [Automata Tutor].
- Join this course (Course ID: 119SWARTH, Course Password: ZDNEIXB5).
- Click “Show” to see the set of practice problems and the set of homework problems.

Each problem will let you attempt several times, and give you useful feedback. You should try the practice problems first, to figure out how to use the site. I recommend that you first try to solve the problems on paper, then use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.

2. Construct a DFA for the language $\{\epsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.

3. Construct a DFA for the language $\{w \mid w \text{ is either } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.

4. Construct a DFA for the language $\{w \mid w \text{ is any string except } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.

5. Construct a DFA for the language $\{w \mid w \text{ contains at least three } 1\text{s}\}$ over alphabet $\Sigma = \{0, 1\}$.

6. Construct a DFA for the language $\{w \mid \text{every } a \text{ in } w \text{ is immediately followed by a } b\}$ over alphabet $\Sigma = \{a, b\}$.

7. Construct a DFA for the language $\{w \mid b \text{ occurs } n \text{ times in } w, \text{ where } n \text{ is divisible by } 3\}$ over alphabet $\Sigma = \{a, b\}$.

8. Construct a DFA for the language $\{w \mid \text{length of } w \leq 5\}$ over alphabet $\Sigma = \{a, b\}$.

9. Construct a DFA for the language $\{w \mid w \text{ contains at least two } 0\text{s and at most one } 1\}$ over alphabet $\Sigma = \{0, 1\}$.

10. **Extra credit.** Construct a DFA for the language $L = \{w \mid w \text{ is a binary number equal to } 1 \mod 3\}$ over alphabet $\Sigma = \{0, 1\}$. (So $0 \notin L$, $1 \in L$, $100 \in L$, etc.)

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1If you want to use late days on this assignment, you will need to submit solutions to these problems via github or on paper. The site has only one deadline, which is 9AM Monday January 30.
Part 2 — These problems should be typeset in \LaTeX and submitted using github.

11. Write a concise English description of the language recognized by DFA $M_1$.

![Figure 1: DFA $M_1$](image1)

12. Write a concise English description of the language recognized by DFA $M_2$.

![Figure 2: DFA $M_2$](image2)

13. Write a concise English description of the language recognized by DFA $M_3$.

![Figure 3: DFA $M_3$](image3)

14. (Sipser 1.41) For languages $A$ and $B$, let the perfect shuffle of $A$ and $B$ be the language:

$$\{w \mid w = \sigma_1 \gamma_1 \sigma_2 \gamma_2 \cdots \sigma_k \gamma_k \text{ where } \sigma_1 \cdots \sigma_k \in A \text{ and } \gamma_1 \cdots \gamma_k \in B \text{ and each } \sigma_i, \gamma_i \in \Sigma\}$$

Prove that the class of regular languages is closed under perfect shuffle. (That is, prove that if $A$ and $B$ are regular, then the perfect shuffle of $A$ and $B$ is a regular language.)