CS41 Lab 7

October 19, 2021

The lab problems this week focus on divide and conquer algorithms. The purpose of this lab is to gain practice using the divide and conquer approach to solving problems.

1. Solve the following recurrence relations (i) using partial substitution, and (ii) using recursion trees.

   (a) $S(n) = S(n-1) + 3n$,
   $S(1) = 3$

   (b) $M(n) = 3M(n/2) + 2n$,
   $M(1) = 1$

   (c) $W(n) = 3W(n/3) + n^2$,
   $W(1) = 1$

   (d) $H(n) = 4H(n/2) + 2n^2$,
   $H(4) = 5$

   (e) $T(n) = 3T(n/3) + 10\sqrt{n}$,
   $T(1) = 5$.

2. **Divide and conquer minimum spanning trees?**

   Lila has a really cool idea for a divide and conquer algorithm which will find a minimum spanning tree. Given a connected, undirected graph $G = (V, E)$ with weighted edges, Lila's algorithm does the following:

   - Divides the graph into two pieces, $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. ($V_1 \cup V_2 = V$ and V_1 and V_2 are disjoint. E_1 is the edges in E with both endpoints in V_1, and E_2 is the edges in E with both endpoints in V_2.)
   - Recursively finds the MSTs M_1 for G_1 and M_2 for G_2.
   - Finds the lowest-weight edge $e = (u, v)$ with $u \in V_1$ and $v \in V_2$.
   - Returns the minimum spanning tree $M_1 \cup M_2 \cup \{e\}$.

   Unfortunately, this algorithm does not work. Give an example input graph G with weights and describe a run of this algorithm where the algorithm does not return a minimum spanning tree on G.